

FUNCTION FITTING TO DEVELOP CLASS PROJECTS: ALCOHOL AWARENESS MODELS IN COLLEGE ALGEBRA

Thomas Edmond Cooper

University of North Georgia

Abstract

In this article, the author shares models that were created for class projects in a section of College Algebra taught as part of a first-year cohort program in which the participating students complete eight introductory college courses as a learning community. Using real-world data and GeoGebra function fitting commands, the author has developed models related to a community engagement project theme of drug and alcohol awareness. Quadratic, rational, exponential, and logistic models are presented along with data and a discussion of how GeoGebra was used to create the models.

Keywords: College Algebra, Modeling, Applications, Quadratics, Exponentials, Rational Functions, Function Fitting

1 INTRODUCTION

In 2013, my university began a new campus in rural north Georgia targeting an underserved population of potential students. As part of this expansion, the administration decided to implement a first-year cohort model, in which participating students take their classes together along with a one hour non-credit leadership course and an out of class community engagement project connecting all of the courses. This instructional model presented the faculty with the challenge of finding ways to demonstrate connections and cohesion between the wide variety of introductory courses found in Table 1. In cooperation with a community partner, we agreed to focus on a community engagement project aimed at drug and alcohol awareness. Out of class, this project involves the students giving presentations in the local schools and participating in events to promote awareness about the dangers of drug and alcohol abuse. In class, we each agreed to create a series of assignments related to the topic.

While I have always tried to give examples of real-world applications in my classes, the College Algebra examples such as projectile motion often seem contrived and unrelated to the students' actual lives. While somewhat challenging, setting out to find alcohol related models with the types of functions that we study in my course has helped me develop a series of assignments that clearly show applications of these functions to a topic familiar to most students and seen as a serious issue in the local community.

Fall Semester	Spring Semester
College Algebra	Introductory Statistics
English Composition I	English Composition II
Introduction to Political Science	Contemporary Global Issues
Introduction to Psychology	Global Ethics

Table 1. First-Year Cohort Courses

An internet search quickly returned examples of linear and exponential models related to alcohol and drug issues (e.g., Mathios, 2012b, 2012a; Sprunt, 2004), but I was less successful finding examples of polynomial or rational models. This is where GeoGebra and function modeling became extremely useful. While my searches failed to turn up explicit models that I could use, I did find an abundance of publications with data and graphical representations. With GeoGebra's function fitting commands, I was able to find accurate real-world models using quadratic, rational, exponential, and logarithmic functions. As of GeoGebra 5.0, there are 11 built in functions for fitting formulas to data. Details of each of these functions can be found under the Statistics category in the online GeoGebra help files (https://wiki.GeoGebra.org/en/Statistics_Commands). The purpose of this paper is to share the models that I developed and discuss how GeoGebra was used to facilitate model determination.

2 QUADRATIC MODELS

I began my search for non-linear models with polynomials. While I could have fit higher ordered polynomials to some of the scatter plots, I did not find any real-world examples with behavior that I feel can be strongly justified by cubic, quartic, or higher degree models. I did, however, find a study of alcohol risk factors with data suggesting quadratic models (Brennan, Schutte, Moos, & Moos, 2011). In this longitudinal study, the authors surveyed participants over 20 years and collected self-reported data on alcohol consumption and drinking problems. Table 2 shows the data from that study for the self-reported average drinks per day.

	Years				
	0	1	4	10	20
Avg. Drinks per day (Men)	1.52	1.41	1.65	1.67	1.10
Avg. Drinks per day (Women)	1.11	1.01	1.15	1.08	0.76

Table 2. Older Men's and Women's Alcohol Consumption over Time

From this data, one can easily derive quadratic models with GeoGebra, using the "FitPoly" command. Below are the instructions, and Figure 1 displays the resulting graphs and models. A GeoGebra file with the data and models can be obtained from <https://www.geogebra.org/m/SfdX9QUc>.

1. Enter the points as lists by typing each of the following in the input bar:

Men = { (0, 1.52), (1, 1.41), (4, 1.65), (10, 1.67), (20, 1.10) }

Women = { (0, 1.11), (1, 1.01), (4, 1.15), (10, 1.08), (20, 0.76) }

2. To find a model for the men enter:
 $m(x) = \text{FitPoly}[\text{Men}, 2]$
3. To find a model for the women enter:
 $w(x) = \text{FitPoly}[\text{Women}, 2]$

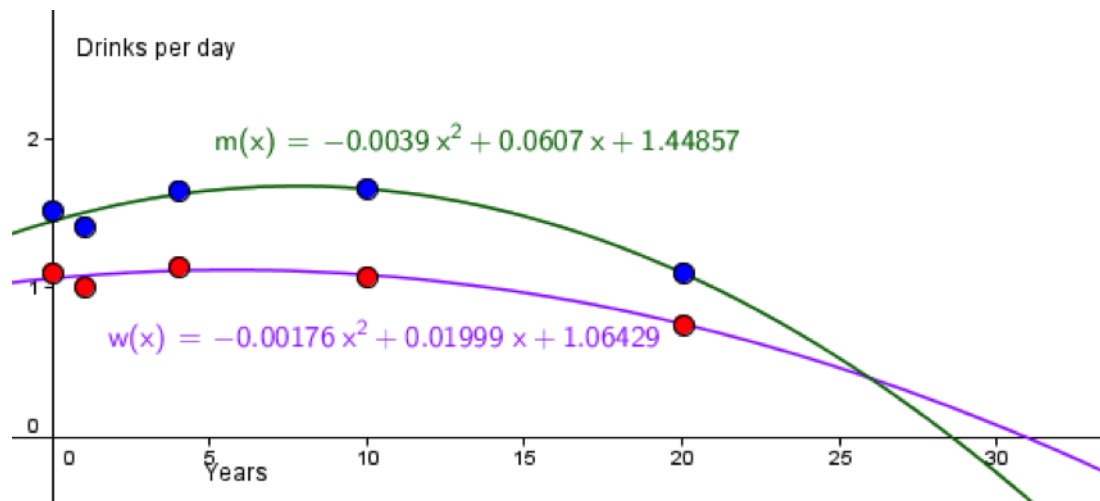


Figure 1. Quadratic Models for Older Men's and Women's Alcohol Consumption over Time

Teachers may wish to have their students find these models themselves or provide the models with questions related to the concepts of quadratic functions studied in an algebra class. I asked my students to graph the functions in GeoGebra and write a detailed explanation of each, including a discussion of the vertex and intercepts. The y -intercepts approximate the average daily consumption of alcohol at the beginning of the study, and the x -intercepts predict when the average consumption would be no daily drinks. Hopefully, students can identify that the x -intercepts, occurring during year 28 for men and year 30 for women, may be extrapolation. The vertex, of course, identifies the time when maximum daily alcohol consumption occurs. This is during the seventh and fifth years after the start of the study for men and women respectively.

Similarly, we can use data on the reported drinking problems, as measured with the Drinking Problems Index (DPI) instrument (Finney, Moos, & Brennan, 1991). Table 3 shows this data, and Figure 2 shows the graphs and models from GeoGebra. The GeoGebra file can be obtained from <https://www.geogebra.org/m/UgQYdSbC>. For these models, I also asked my students to write detailed descriptions and interpretations, including a discussion of the vertex and intercepts. Here the y -intercepts approximate the participants' average DPI scores at the beginning of the study. The models do not have x -intercepts due to the average DPI always being positive, i.e., there were always some measurable drinking problems among the participants. For the males, the DPI decreases over time, and the vertex is clearly extrapolation, occurring after 20 years. For females, the vertex is not extrapolation, occurring around the 14th year; however, the female DPI scores are strictly decreasing with the model's minimum and the data values at 10 and 20 years differing by only a few one-hundredths of a point. Therefore, while the quadratic models provide accurate fits to the data,

the data does not support an argument that the DPI scores are parabolic with a point where they will eventually begin to increase over time.

	Years				
	0	1	4	10	20
Drinking Problems (Men)	1.34	1.29	1.11	0.83	0.57
Drinking Problems (Women)	0.83	0.59	0.47	0.30	0.28

Table 3. Drinking Problem Index for older men and women over time

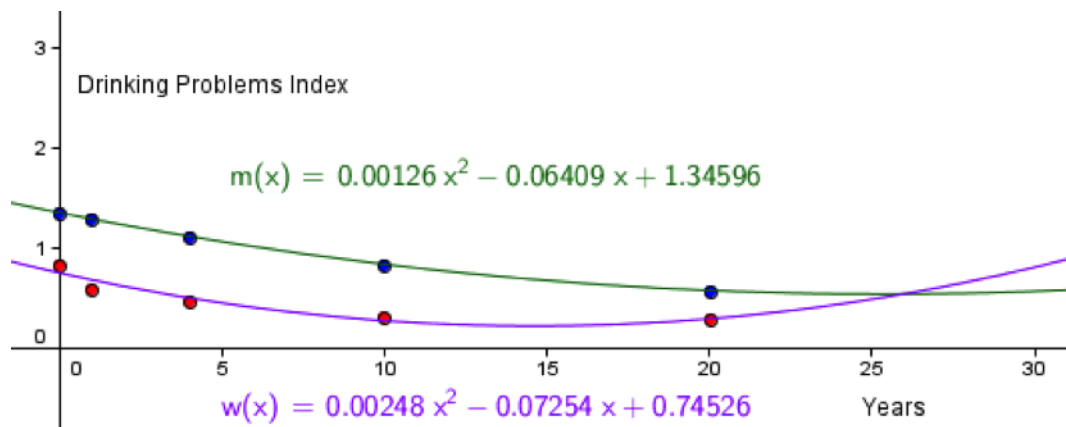


Figure 2. Model of Drinking Problem Index for older men and women over time

3 RATIONAL MODELS

Following polynomials, my College Algebra class turns to rational functions and their graphs. As with quadratics, I searched online for data or graphs that could be explained with these types of functions. My search led me to two types of graphs that I was able to model quite accurately with rational functions using GeoGebra and the general “Fit” command.

3.1 Relative Mortality Risk

In the British Columbia Medical Journal, Andrade and Gin (2009) present data on the relationship between alcohol consumption and an individual’s relative mortality risk (RMR). This numerical index compares an individual’s mortality risk to the rest of the population. A relative mortality risk of 1, indicates that the individual has the same risk as the population. A value above 1, indicates a higher risk than the population. In the article, the authors present the data for men and women as line graphs which they describe as “J-shaped curves”. For my class projects, I estimated the values from the graph as shown in Table 4. An accurate model for this type of data needs to have a y -intercept of $(0, 1)$ to match the risk for the general population and a slant asymptote to model the end behavior. Thus I decided to use models of the form

$$y = \frac{ax^2 + bx + c}{dx + c}.$$

Drinks per day	0	0.5	1	1.5	2	2.5	3	3.5	4
Men	1.00	0.81	0.84	0.86	0.88	0.92	0.96	1.00	1.06
Women	1.00	0.82	0.86	0.91	0.99	1.06	1.14	1.25	1.39

Table 4. Men's and Women's Relative Mortality Risks versus Average Drinks per Day

Below are instructions to find the models in GeoGebra, and the results appear in Figures 3a and 3b. Both models are in the GeoGebra file available at <https://www.geogebra.org/m/sqtkDj4v>.

1. Enter the points as lists by typing the following in the input bar:
 Men={ (0, 1), (.5, .81), (1, .84), (1.5, .86), (2, .88), (2.5, .92), ... }
 Women={ (0, 1), (.5, .82), (1, .86), (1.5, .91), (2, .99), (2.5, 1.06), (3, 1.14), (3.5, 1.25), (4, 1.39) }
2. Create sliders named a , b , c , and d . Preexisting sliders are a requirement of the "Fit" command. The specific slider settings apparently do not matter. The GeoGebra tutorial recommends the default settings of -5 to 5 by increments of 1 .
3. To find a model for the men enter:
 $m(x) = \text{Fit}[\text{Men}, (a \cdot x^2 + b \cdot x + c) / (d \cdot x + c)]$
4. To find a model for the women enter:
 $w(x) = \text{Fit}[\text{Women}, (a \cdot x^2 + b \cdot x + c) / (d \cdot x + c)]$

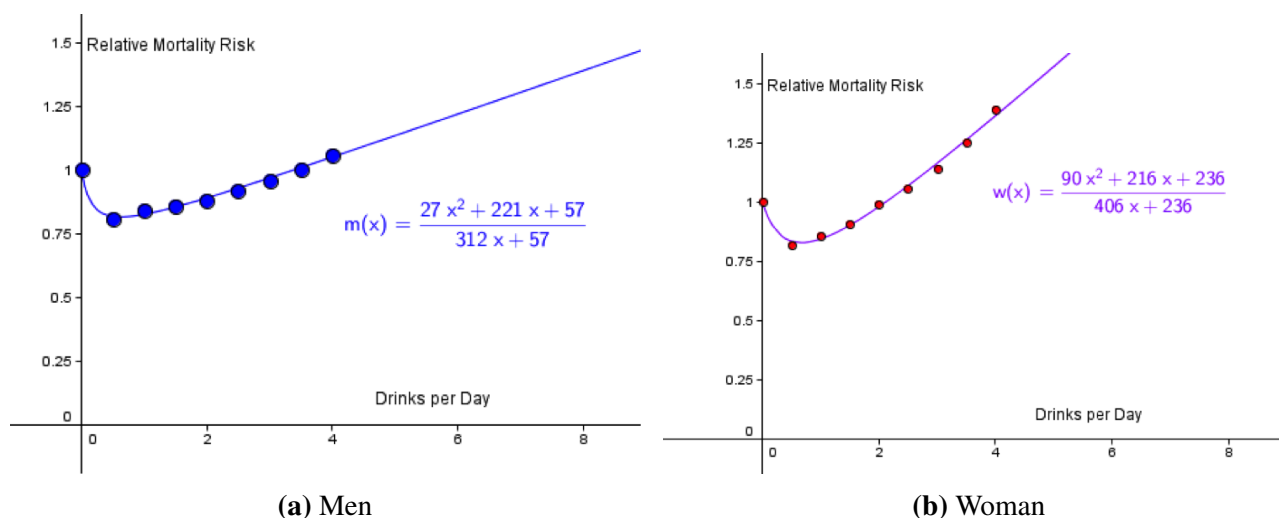


Figure 3. Relative Mortality Risk versus Average Drinks per Day

For my College Algebra assignments, I provided the functions and asked the students to complete the following:

- (a) For each function, find and interpret the y-intercept.
- (b) Find the slant asymptote for each function, showing all work.
- (c) Using GeoGebra, plot both functions on the same graph.
- (d) For each function, find the minimum value.
- (e) For each of the two functions, write a few sentences to describe what the functions and their graphs tell us about alcohol and the individual's relative mortality risk. Be sure to include all

important aspects of the graphs and functions.

As noted, the students should observe that the y -intercepts are necessarily zero because a person consuming zero average drinks per day should have the same RMR as the general population. The RMR scores are also necessarily positive, and thus there are no x -intercepts. Interestingly, the shape of the graph indicates that some alcohol consumption actually leads to a decrease in RMR with minimums occurring around 0.6 to 0.7 drinks per day. For men, the RMR actually remains below 1.0 until 3.5 drinks per day, but then steadily increases asymptotically. For women, the RMR exceeds 1.0 by 2.5 drinks and then increases at a faster rate than the men.

3.2 Peak BAC (A Surge Function)

Simple linear models for blood alcohol content (BAC) do not take into account the fact that alcohol is not immediately absorbed by the body. Research shows that BAC follows a “surge” pattern where the level climbs to a peak and then starts to decrease back toward zero if no other alcohol is consumed. Using a graph from Barr (1968), I approximated data points as shown in Table 5. An appropriate model for this function needs to pass through the origin, peak, and decrease asymptotically toward the x -axis. After some trial and error, I used the GeoGebra command:

`Fit[data, (a*x^2+b*x) / (c*x^4+d*x^2+e)]`

where “data” is the list of point and the coefficients have been previously defined as sliders. The scatter plot along with the best fit function of this form can be seen in the GeoGebra screenshot in Figure 5. The GeoGebra file can be obtained from <https://www.geogebra.org/m/KM4y2T5U>.

Hours	0	0.5	1	1.5	2	3	4	6	8	10	12
BAC	0	0.064	0.080	0.081	0.079	0.070	0.060	0.044	0.032	0.022	0.018

Table 5. Blood Alcohol Concentration (BAC) over time

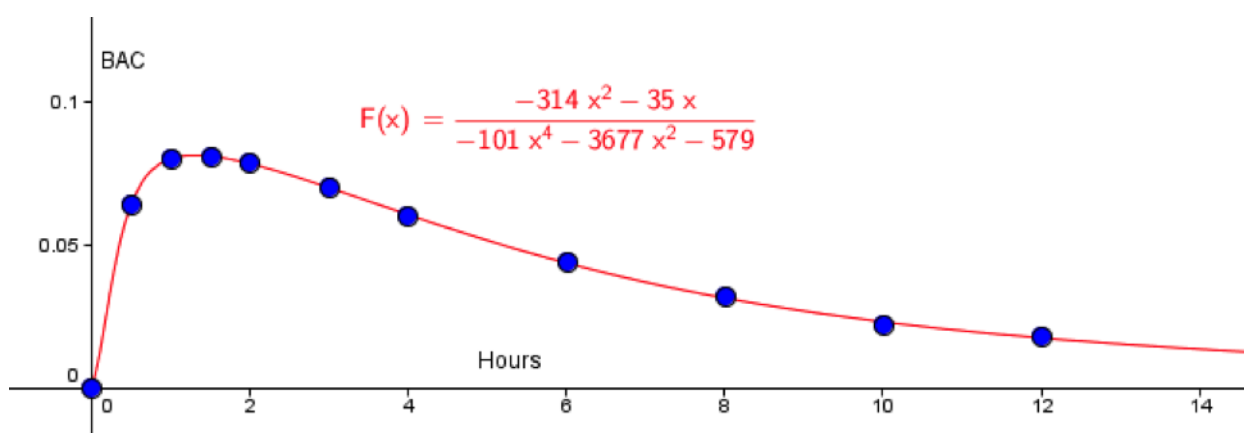


Figure 4. A Rational Model for Blood Alcohol Concentration versus Time

Similar to the risk models tasks, I provided my students with the model and asked the following questions:

- (a) Find and interpret $F(0)$.
- (b) Explain a reasonable domain and range for this function.
- (c) Find the horizontal asymptote and explain what this tells us about this person's BAC.
- (d) Use software such as GeoGebra to create a graph of this function, keeping in mind the domain and range from part (b).
- (e) Using the graph, determine when the peak (maximum) BAC level occurs.
- (f) Using the graph or table, determine when this person's BAC will fall below 0.04.

As noted, students should conclude that $(0, 0)$ is necessarily an x - and y -intercept because this represents no alcohol in the system prior to consumption. Students should also observe from the overall shape that the BAC increases rapidly to a peak level of approximately 0.08, the legal driving limit in the United States, between 1 and 2 hours and then decays toward zero. The horizontal asymptote at $y = 0$, did reveal a surprising misconception among several of my students, who claimed that this meant that there would always be some miniscule amount of alcohol present in the body. Apparently, these students fixated on my theoretical description of such an asymptote, and these responses opened the door to a discussion of the limitations of simplified models applied to the real world. While it is difficult to give an exact upper bound on the domain, students should understand that the amount of alcohol in the body is initially zero and will at some point return to that amount.

4 EXPONENTIAL MODELS

My College Algebra class concludes with a chapter on exponential and logarithmic functions. To explore these topics, I revisited decreasing BAC over time and modeled data related to drinking and driving. The models and tasks that I developed are discussed in the remainder of this section.

4.1 Exponential BAC over Time

If one only considers the times after peak BAC has been reached, the data closely follows an exponential decay function. This too can easily be modeled with GeoGebra using the command `FitExp[data]`. Where the data contains the eight data points in Table 5 from 1.5 hours to 12 hours. Figure 5 shows a screenshot of this model with the scatter plot. The GeoGebra file can be obtained at:
<https://www.geogebra.org/m/nePHxh5M>.

On this assignment, I had two main goals for my students. I wanted them to explore the use of a real-world exponential functions, including the use of logarithms, and I wanted them to compare the exponential and rational models. For the latter, they observed that the rational model is required for accurately representing all of the data, but the exponential model is easier to use when working with the points beyond the peak. For the former, I asked them to use the model, solving by hand with logarithms, to determine when this person's BAC would fall below 0.05, and I asked them to find and discuss the inverse function, $y = -6.6181 \ln(9.3633x)$. I expected a simple interpretation that inputting a BAC between 0 and 0.08 reveals the amount of time needed to reach that level, but a

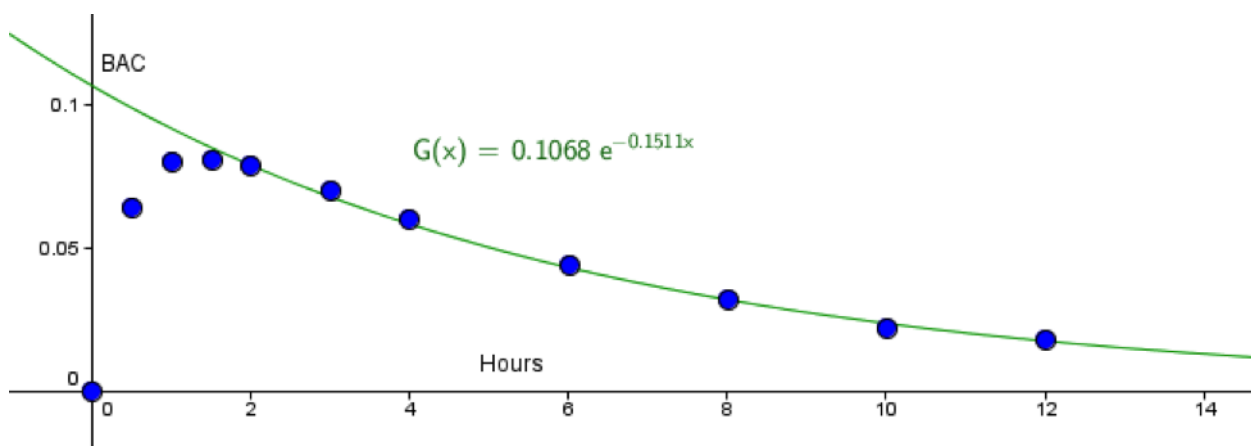


Figure 5. An Exponential Model for Blood Alcohol Concentration versus Time

few students found the decreasing graph with BAC on the horizontal axis and time on the vertical axis difficult to comprehend. These students questioned why the graph seems to show that as BAC increases, time decreases. Thus, this problem presented another unanticipated opportunity to discuss the implications of a real-world context. Despite the appearance of the conventional graph of the inverse function, in our problem, BAC does not increase over time. It decreases from the peak level toward zero.

4.2 Drinking and Driving

A major goal of our community engagement project was to discourage driving under the influence of alcohol. A research paper by Preusser (2002), provides excellent data for exploring the relationship between BAC and fatal car crashes. Table 6 shows the data by age group relating BAC to the relative fatal crash rates which Preusser obtained from a crash database from 1987 to 1999 in the United States. As with the other relative risk examples, a rate of 1 means that a person has the same risk as a randomly selected member of the general population. A relative risk rate of 2 would mean that the person is twice as likely to be involved in a crash, and so on.

		BAC							
Age	0	.01	.02-.03	.04-.05	.06-.07	.08-.09	.10-.14	.15-.19	≥0.20
16-20	3.31	4.37	4.12	5.44	8.17	10.10	15.77	25.30	28.19
21-24	1.79	2.18	2.59	4.42	6.11	8.13	10.73	16.43	26.00
25-34	1.25	1.38	1.89	2.32	2.94	4.37	7.27	11.61	16.08
35-49	1.00	1.09	1.49	1.78	2.62	3.56	5.64	10.44	16.99
50-64	1.02	0.93	1.17	1.24	2.03	2.23	4.71	8.48	13.24
65+	2.04	1.97	2.49	2.50	2.50	3.55	4.83	7.48	9.48

Table 6. Relative Fatal Crash Risks by Age Group and BAC

For the quadratic and rational projects, I decided that it would be better to provide the students with the models and focus the assignments on using and interpreting these models. My reasoning for this decision was that regression and model fitting are not a direct topic of the course, and the students were unexperienced with GeoGebra at that time. I did not want this extra task to hamper the other

parts of the assignments. For the final project, I decided to let the students find the exponential models for themselves. By this point in the semester, they were more comfortable with GeoGebra and were easily able to follow the instructions to use the `FitExp` command. Below are the questions that I posed to my students.

- Write a few sentences explaining what the information in the column for a BAC of 0 tells us about fatal crashes and age.
- Using GeoGebra, plot the BAC versus relative risk rates for the 16-20 year olds, using the average when a BAC range is given and using a BAC of .2 for the last column: (0,3.31), (.01, 4.37), (.025, 4.12), (.045, 5.44), (.065, 8.17), (.085, 10.10), (.12, 15.77), (.17, 25.30), (.2, 28.19)
- Add the best fit exponential function to the graph (using the `FitExp` command).
- On the same graph, plot the BAC versus relative risk rates for the 35-49 year olds, using the average when a BAC range is given and using a BAC of .2 for the last column.
- Add the best fit exponential function for the 35-49 year olds' data.
- Using algebra to solve and showing your work, at what BAC would a 16-20 year old have 10 times the risk of being involved in a fatal crash?
- Using algebra to solve and showing your work, at what BAC would a 35-49 year old have 10 times the risk of being involved in a fatal crash?
- Write a paragraph describing what these two graphs tell us about BAC and the risk rates for the two groups. Be sure to compare and contrast the two groups.

Figure 6 shows the two exponential models along with the data points. The function $g(x)$ represents the 35 – 49 year olds, and $f(x)$ represents the 16 – 20 year olds. For both age groups, the risk of being involved in a fatal car crash increases exponentially as the BAC increases, and the risk is always higher for the younger drivers. The GeoGebra file for these models can be obtained at <https://www.geogebra.org/m/jQpNkbYW>.

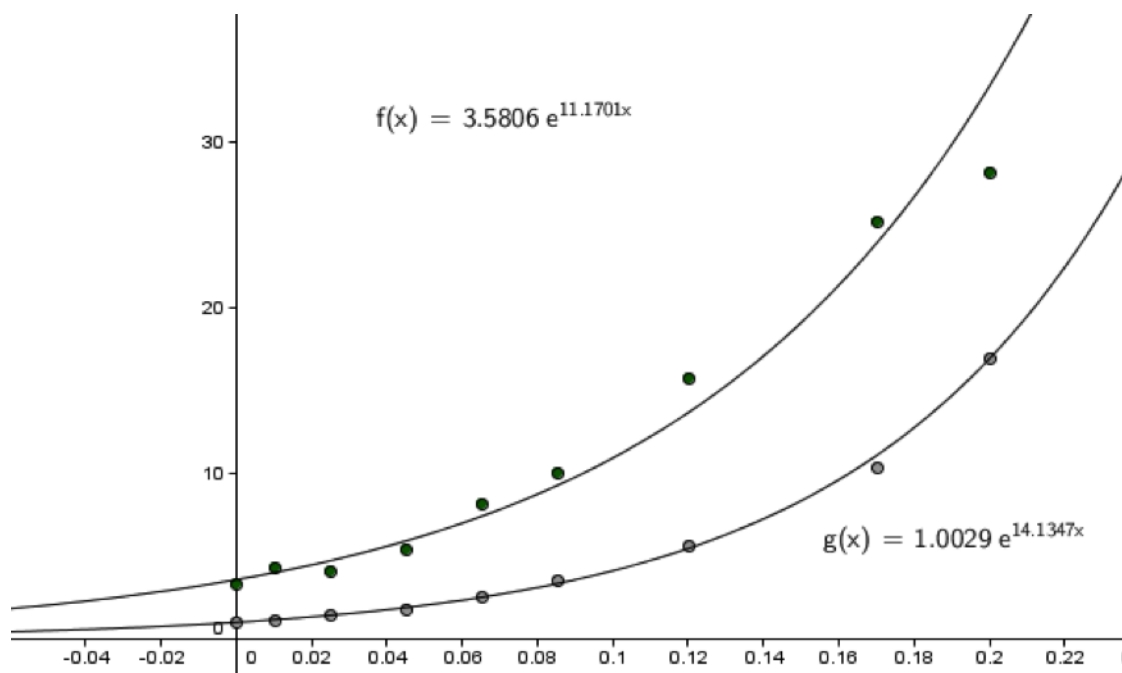


Figure 6. Relative Risk of Fatal Crash versus BAC by Age

5 LOGISITIC MODELS

Another group of researchers (Compton et al., 2002) used data from Long Beach and Fort Lauderdale to compute relative risk rates for fatal crashes, not separating the data into age groups.

A total of 2,871 crashes were sampled (1,419 in Long Beach and 1,452 in Fort Lauderdale). Overall, 14,985 drivers were approached for participation in the study. There were 4,919 crash (2,422 in Long Beach and 2,497 in Fort Lauderdale) and 10,066 control drivers 5,006 in Long Beach and 5,060 in Fort Lauderdale (Compton et al., 2002).

Table 7 contains the data from this study. Like the previous example, I chose to have my students find the model themselves with GeoGebra.

BAC	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
Relative Risk	1.00	1.03	1.03	1.06	1.18	1.38	1.63	2.09	2.69
BAC	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17
Relative Risk	3.54	4.79	6.41	8.90	12.60	16.36	22.10	29.48	39.05
BAC	0.18	0.19	0.20	0.21	0.22	0.23	0.24	0.25	
Relative Risk	50.99	65.32	81.79	99.78	117.72	134.26	146.9	153.68	

Table 7. Relative Risk of Fatal Crash by BAC

Below are the questions that I posed for this assignment.

- Use GeoGebra to plot this data. Be sure to use an appropriate viewing window.
- Use the command `FitLogistic` to fit the data with a logistic model.
- According to the model, what is the relative risk of being involved in a fatal car crash for a driver with a BAC of 0.08? How does this compare to the actual data?
- Using the model, determine what BAC would give a driver a relative risk of being involved in a fatal car crash of 20. Show your work, using logarithms as appropriate.
- Write a few sentences describing what this logistic function and its graph tell us about BAC and relative risk of being involved in a fatal car crash. As always, be sure to include all of the important features of the graph.

Figure 7 shows the logistic model along with the data. The GeoGebra file can be obtained from <https://www.geogebra.org/m/vqbhntnb>. As with the previous model, the relative risk of being involved in a fatal car crash increases rapidly as a person's BAC rises, but the logistic model shows a horizontal asymptote at $y = 191$. Of course, there is a physical limit on the domain of an individual's BAC, as levels above 0.30 lead to unconsciousness or death.

6 CONCLUSION

Being faced with the requirement of building cohesion into the cohort program by creating assignments related to our community project of drug and alcohol awareness, I was able to use GeoGebra's function fitting commands to find several real-world models based on published research data. These

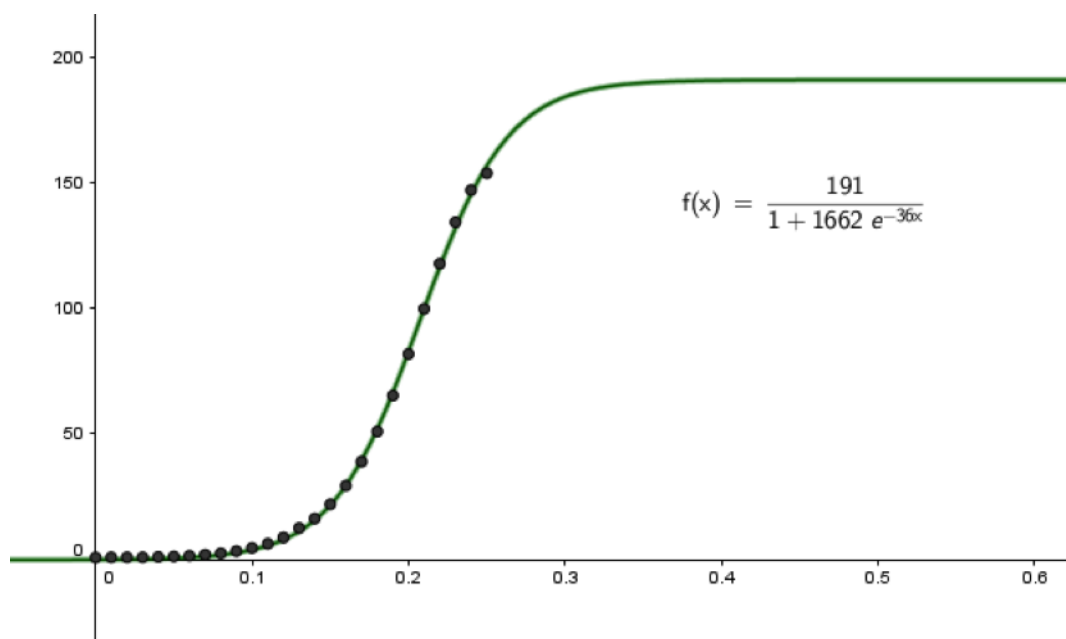


Figure 7. Graph of Logistic Model of Relative Risk of Fatal Crash versus BAC

models include quadratics, rational functions, exponentials, and logistic functions, all common topics in high school and introductory college courses. While I have yet to collect empirical data to address the impact of these assignments, I am confident that the students completing these projects have a much higher awareness of the applicability of the course content to real-world situations. By the end of the second semester, these students will have used these assignments along with those from other courses to create and present materials to discourage underage drinking and substance abuse at the local high school. Hopefully, this experience will greatly increase their appreciation for the usefulness of the course content and mathematics in general.

For my assignments, my primary goal was not to teach the process of mathematical modeling. I was more concerned with finding functions grounded in the contextual setting of alcohol awareness to use with traditional algebra procedures such as equation solving and exploring the key features of elementary functions by hand. GeoGebra, however, makes the process of converting from numerical data to symbolic functions rather simple with the built in function fitting commands. Therefore, I will definitely add this task to all future assignments. Additionally, I believe that I can find more ways to for students to benefit from GeoGebra's dynamic linking of multiple representations. For example, I had to define sliders as a requirement to implementing the "Fit" command with rational functions, but I did not explicitly use the sliders myself or require the students to do so. It would certainly be a valuable experience to have the students plot the data, define sliders, and use them to control functions in order to dynamically find close fitting functions before using the built in commands.

I encourage readers teaching courses with the functions presented in this article to try out these applications, and I hope that the examples can inspire others to find their own models from other data to build additional real-world applications for students to explore. The challenge is finding meaningful data that can be modeled with functions used in our classes, but once such data is acquired GeoGebra's function fitting commands make finding the models quick and simple. With a little effort, we can create much more meaningful examples and exercises for our students than the contrived word

problems often seen in traditional textbooks.

REFERENCES

- Andrade, J., & Gin, K. G. (2009). Alcohol and the heart. *Alcohol*, 51(5).
- Barr, W. (1968). Principles of biopharmaceutics. In *American journal of pharmaceutical education* (Vol. 32, p. 958).
- Brennan, P. L., Schutte, K. K., Moos, B. S., & Moos, R. H. (2011). Twenty-year alcohol-consumption and drinking-problem trajectories of older men and women. *Journal of studies on alcohol and drugs*, 72(2), 308–321.
- Compton, R. P., Blomberg, R. D., Moscovitz, H., Burns, M., Peck, R. C., & Fiorentino, D. D. (2002). Crash risk of alcohol impaired driving. In *Proceedings international council on alcohol, drugs and traffic safety conference* (Vol. 2002, pp. 39–44).
- Finney, J. W., Moos, R. H., & Brennan, P. L. (1991). The drinking problems index: A measure to assess alcohol-related problems among older adults. *Journal of substance abuse*, 3(4), 395–404.
- Mathios, D. (2012a). *Math 212 drinking and driving project exponential version*. Retrieved 2017-04-20, from https://www.deanza.edu/faculty/mathiosdiane/pdf/blood_alcohol_level_project_exponential.pdf
- Mathios, D. (2012b). *Math 212 drinking and driving project linear version*. Retrieved 2017-04-20, from https://www.deanza.edu/faculty/mathiosdiane/pdf/blood_alcohol_level_project_linear.pdf
- Preusser, D. F. (2002). Bac and fatal crash risk. In *Proceedings of the 16th international conference on alcohol, drugs and traffic safety* (pp. 4–9).
- Sprunt, H. (2004). *Blood alcohol concentration: Modeling with mathematics*. Retrieved 2017-04-20, from http://www.appstate.edu/~spruntwh/bac_ncctm.pdf



Tom Cooper, teaches mathematics and mathematics education courses at the University of North Georgia. He has degrees in mathematics from the University of Tennessee and a Ph.D. in mathematics education from the University of Georgia. His research interests include investigating the use of student-centered teaching methods and using technology to develop conceptual understanding.