MODELING A TOTAL SOLAR ECLIPSE USING GEOGEBRA

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Abstract

Using a modeling approach, this article discusses how to build a GeoGebra model to help students make sense of a total solar eclipse. A few powerful tools in GeoGebra such as sliders, tangents, and animation allow students to explore the mathematical dynamics of a solar eclipse and identify the causes behind a total or partial solar eclipse.

Keywords: Modeling, solar eclipse, sense making

1 INTRODUCTION

On August 21, 2017, a spectacular astronomical event will be witnessed across the continental United States–the 2017 total solar eclipse, for the first time in the 21st century [5]. From Oregon to Georgia, the shadow of the Moon will race through a large part of the country in about 1 hour and 33 minutes. Along the path, the total solar eclipse will last up to two minutes and 40 seconds, depending on the specific location of observation. This is a life-changing event, according to astronomers and those who had experienced a total solar eclipse [5]. There is an ongoing effort to engage the numerous communities and K-12 schools along the path in preparing for this unique event in terms of safety, education outreach, and various social and cultural implications of a total solar eclipse. The 2017 total solar eclipse is certainly an exciting opportunity for mathematics and science educators across the grade levels. There is no doubt that the vast majority of students have not experienced a total solar eclipse in their lives. They will have numerous questions about the Sun, the Moon, the Earth, and the universe at large while anticipating this natural spectacle. Certainly, there are a host of scientific facts and folklore about the total solar eclipse that are worth exploring in a K-12 classroom. There is also an enormous amount of school mathematics behind this type of astronomical event, including measurement, geometry, and algebra [7, 9].

This article demonstrates the key mathematical processes of a total solar eclipse using the technological features of *GeoGebra*, a free and open modeling software environment accessible to classroom teachers around the world. Our overarching goal is to make sense of this fascinating astronomical phenomenon from a K-12 mathematical standpoint and showcase the pedagogical power of *GeoGebra* in making connections between mathematics and physical sciences. According to our field experiences, a great many of inservice and preservice elementary teachers have heard about a solar eclipse but are not familiar with the mathematical or physical processes behind it. There is also a crying need to bring models and modeling into the practices of mathematics and science teaching [4]. Therefore, we encourage our readers and educators to pre-assess the existing knowledge of their audience about a solar eclipse in a non-threatening manner, before tackling the following tasks using *GeoGebra*.

2 FORMATIVE ASSESSMENT

The Sun, the Earth, and the Moon are all naturally familiar to young students as well as their teachers. However, the mathematics behind a solar eclipse remains an uncharted domain for most of them, children and adults alike, mainly because of the lack of prior exposures. Therefore, it is pedagogically advisable to assess the existing knowledge of a targeted audience before any instruction or intervention. There are a variety of questions that can be asked about these three major bodies such as their sizes, distances, orbits, and inclinations. These descriptors, however, only make sense with respect to a conceptual model. Therefore, it is more meaningful to assess the existing mental models of students in the context of solar eclipses. In a teaching experiment in spring 2016, we asked a group of 27 elementary teacher candidates to respond to the following open-ended prompt: *How does a total solar eclipse happen? Please sketch a model and then explain to the best of your knowledge.* Two types of data were collected: verbal explanations and visual models, both of which helped us make sense of their prior knowledge of a solar eclipse. Four of the 27 participants reported having no idea about a solar eclipse. The rest demonstrated a variety of misconceptions or incomplete understandings, which can be concluded on the basis of both verbal explanations and visual sketches.

Verbally, our participants were struggling to grasp the relationships among the Sun, the Moon, and the Earth, as is evidenced in the following sample explanations (TC for Teacher Candidate):

TC 1: A total eclipse occurs when the Earth blocks sunlight from the Moon.

TC 2: The Sun covers the Moon.

TC 3: The Moon and clouds cover the Sun.

TC 4: I think it is when the Moon covers the view of the Sun from the Earth.

TC 5: I believe it's when the Sun and the Moon are aligned, and the Moon is blocking the Sun. I don't exactly know how that happens though.

TC 6: The entire Sun is blocked out by the Moon leaving a black sky for a short time.

The majority of the teacher candidates were aware of the potential interactions among the Sun, the Moon, and the Earth but were uncertain about their mathematical or physical relationships, as can be further seen in their visual models. Figures 1, 2, and 3 are three samples of our participants' visual models for a solar eclipse, which speak about their conception about the Sun, the Moon, and the Earth. The drawings are full of details. The Sun has light radiations; the Moon has craters; and the Earth has continents. Further, the three bodies are aligned during a solar eclipse such that the Moon blocks sunlight from reaching the Earth. However, there are two essential components missing in almost all the models: the scales and the dynamic processes, which can be investigated using *GeoGebra*.

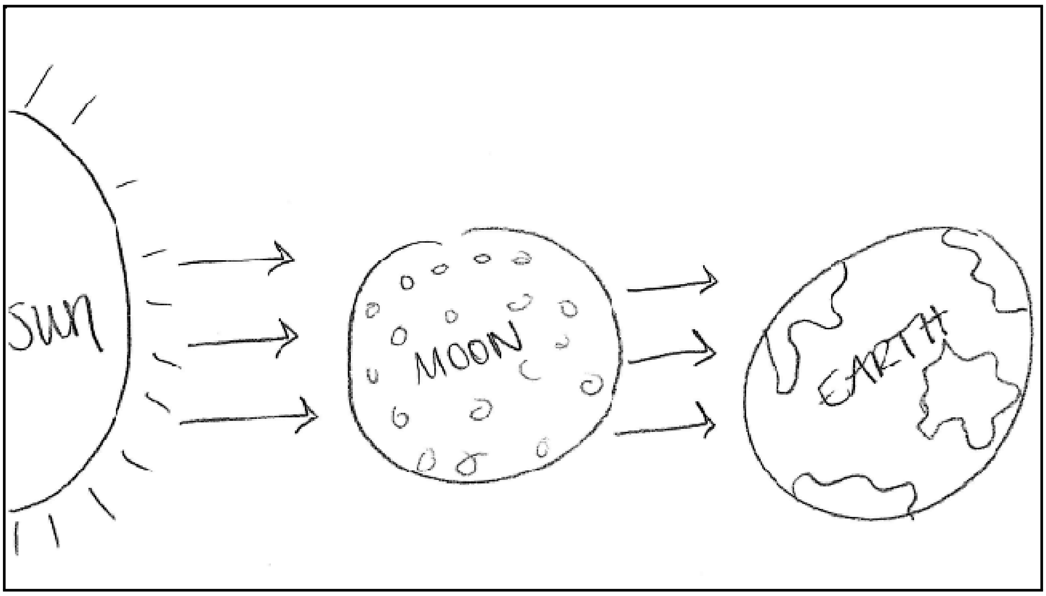

Figure 1. A teacher candidate's visual model showing the alignment of the Sun, the Moon, and the Earth.

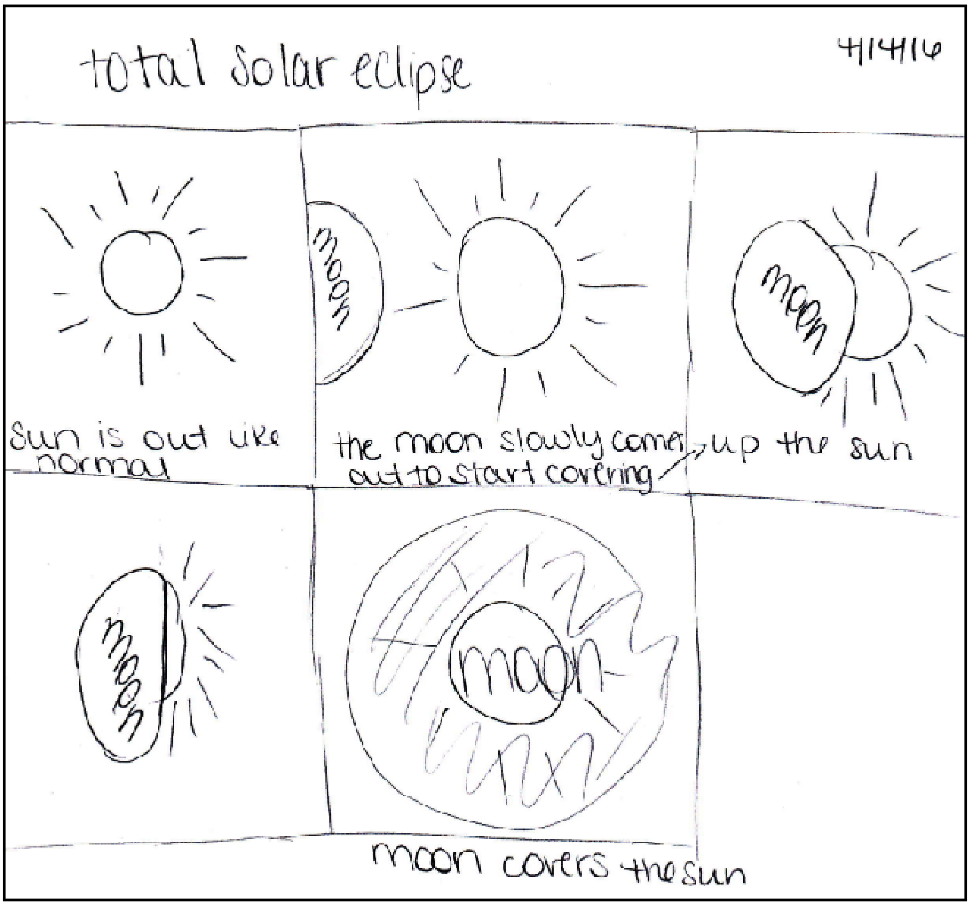

Figure 2. A teacher candidate's visual model showing the progression of a solar eclipse.

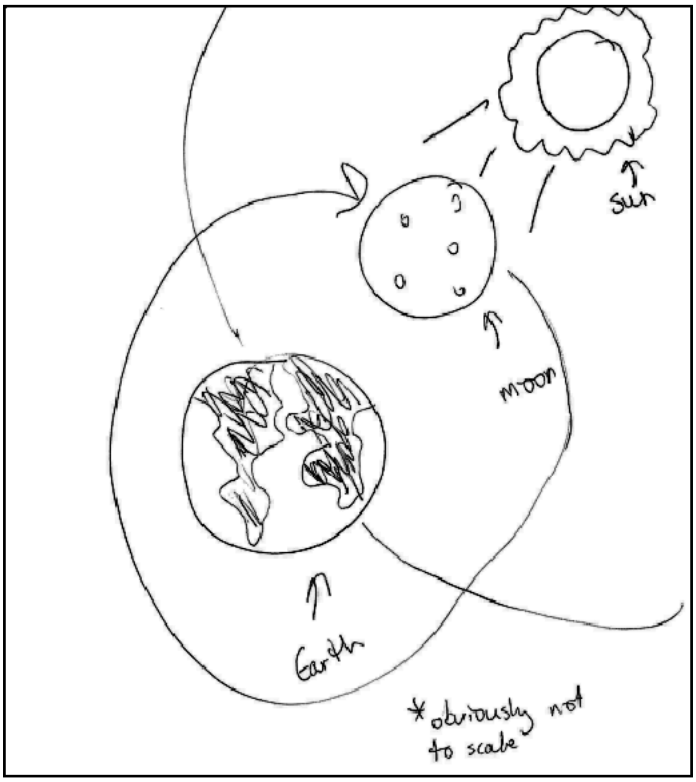

Figure 3. A teacher candidate's visual model showing the alignment of the Sun, the Moon, and the Earth and their orbits.

3 ABOUT SCALES AND PROPORTIONS

When modeling astronomical bodies, we have to deal with scales and proportions [1, 2]. Because of the dramatic differences in sizes and distances, it is virtually impossible to scale everything down to a computer screen while maintaining the proportionality. For example, the Moon has a diameter of about 3,476 kilometers. By contrast, the Sun has a diameter of about 1,392,000 kilometers. That is a ratio of about 1:400. If the Moon has a diameter of 6 pixels on a computer screen, the Sun will take over the whole width of even a high-resolution screen. Furthermore, there is the ratio between the distances. The Moon is about 384,400 kilometers from the Earth on average. The Sun is about 150,000,000 kilometers from the Earth on average. The ratio is about 1:390. These two ratios are close, contributing to the frequency and coverage of a solar eclipse [1].

For the purpose of mathematical exploration, therefore, we need to use different scales to make sense of the key processes of a solar eclipse. For students in the middle and secondary grades, we can consider the following design task using *GeoGebra* (Figure 4). In the *GeoGebra* Geometry Window, we construct two circles centered at A and B with a radius of 4 units and 1 unit, respectively. Any other reasonable radii can be used. Using the gridlines as a reference, we move points A and B to the same horizontal line. Further, we place a third point C on the same line. The relative distances between A, B, and C can be adjusted as the student wishes. Next, we construct two tangent lines from point C to one of the circles, say, the circle centered at B. We further construct the points of tangency K and J, as shown in Figure 4. Then, we move point A along the line until the circle centered at A is also tangent to the two existing tangent lines. We further construct the points of tangency F and H.

To explore the proportional relations in this configuration, we make segments AF and BK. Finally, we can pose the question about the distances. If AF is 4 units and BK is 1 unit, what is ratio between CB and CA and why? Other questions can be subsequently posed for students to understand the structure of the problem. For example, what if we change the relative distances between A, B, and C while still keeping the tangencies? How about the ratio between the area of triangle BCK and that of triangle ACF? What if we change the relative sizes of the circles? Through these questions and *GeoGebra*-enabled explorations, students could gradually come to understand the implications of similarity in this context in preparation for the next steps.

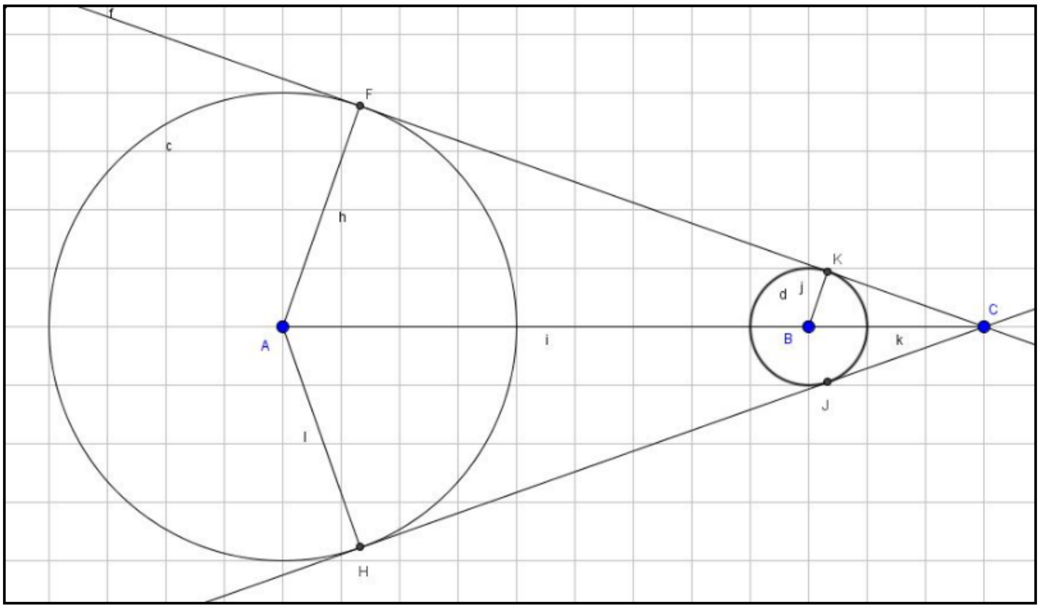

Figure 4. Exploring scales and proportions.

4 Setting up the Stage

To make sense of a total solar eclipse, we now set up the stage and construct three circles with variable radii (Figure 5). *GeoGebra* sliders are powerful pedagogical tools for students to visualize independent variables, leaving room for future manipulations. For the current project, we define three variables, visualize them as sliders, and change their properties as needed – `RadiusSun`, `RadiusEarth`, and `RadiusMoon`, which are initialized to 4, 2, and 0.5 units. We then construct three circles with the above radii for the Sun, the Earth, and the Moon, respectively, centered at points S, E, and M one the same horizontal line. The relative distances between S, E, and M are not important in the beginning. However, the Moon should be closer to the Earth than the Sun. These distances can be manipulated later on for open explorations. Finally, the `zoom in` and `zoom out` features of *GeoGebra* can be used to scale the screen objects for visual clarity.

5 CONSIDERING SOME POINTS ON THE SUN

With the major players set up in Figure 5, we can now consider one arbitrary point on the surface of the Sun, such as point P in Figure 6. From point P, there are infinitely many light rays radiating into the space in straight lines (for our purpose), some of which will reach and light up the surface of the

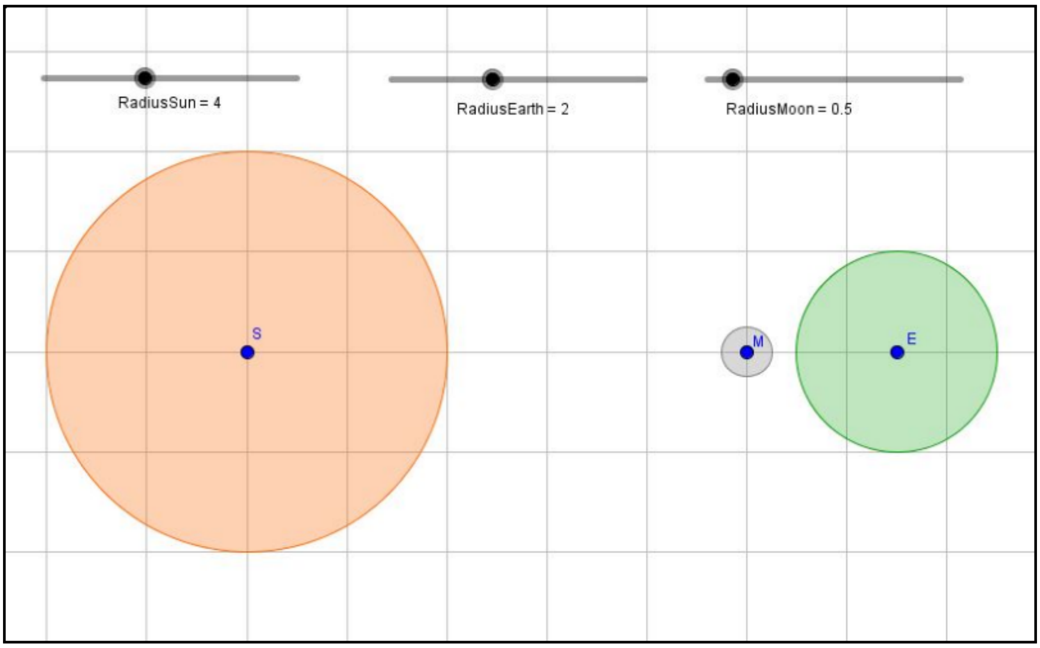

Figure 5. Setting up the stage for modeling a solar eclipse.

Earth while others will be blocked by the Moon if the Moon is in between the Sun and the Earth. Our question is which part of the Earth cannot receive light rays from point P? To address this question, we need to construct the two tangent lines from point P to the Moon, which help delimit the shadow of the Moon in this special case (Figure 6). A person standing in the shadow thus cannot see point P because the point is blocked by the Moon.

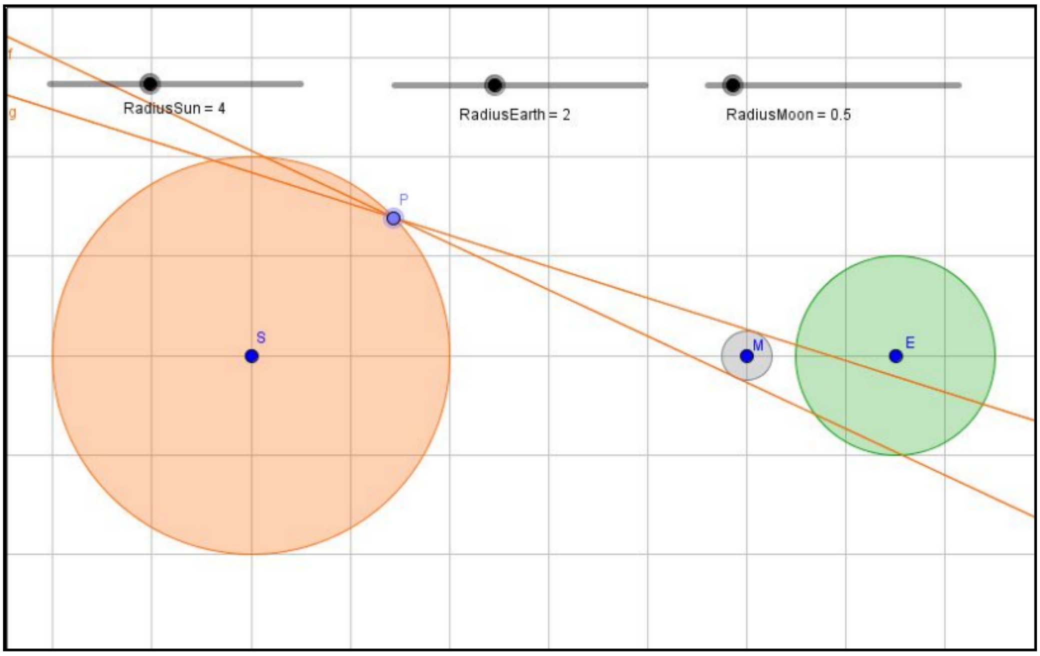

Figure 6. Modeling one point on the surface of the Sun and the corresponding shadow of the Moon on the Earth.

In Figure 6, Point P is, of course, a theoretical point, which is useful for formulating an explanation. The Sun has infinitely points on its surface, each of which is radiating light rays toward the Earth. We could therefore pick more points and repeat the above thought experiment in order to find a pattern. For example, Figure 7 is a typical illustration used to explain a total eclipse [6], which is straightforward to an expert but seems mysterious to novices. To fill up the gaps, we resort to *GeoGebra's* tracing tool to demonstrate the dynamic processes of a total solar eclipse.

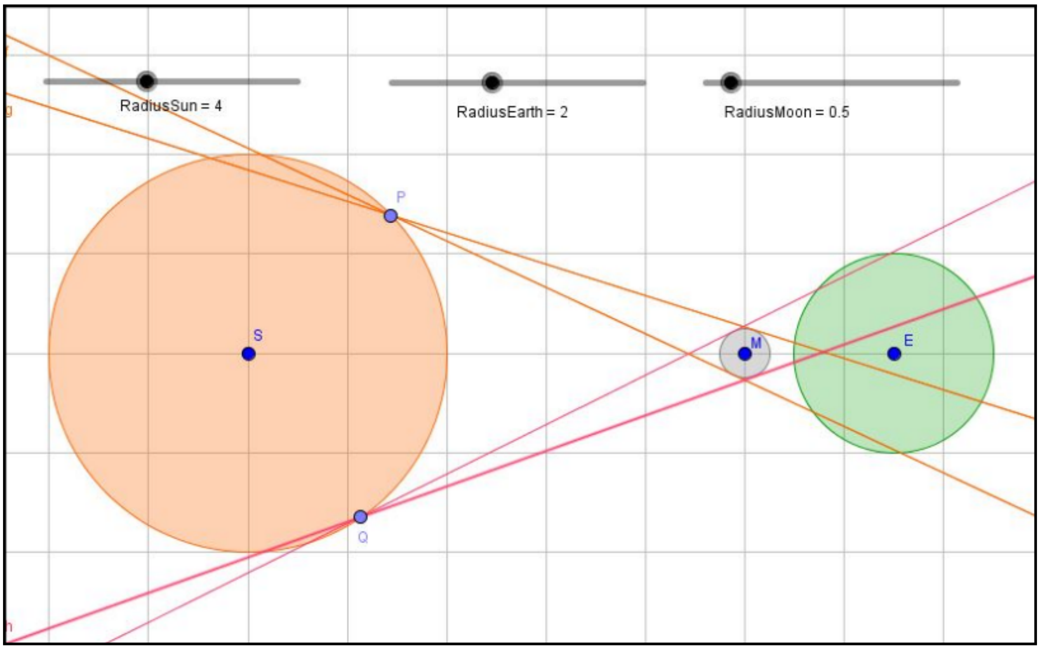

Figure 7. Modeling two points on the surface of the Sun and the Moon's overlapping shadows on the Earth.

6 MAKING SENSE OF A TOTAL ECLIPSE

In Figure 6, we considered a special point P on the surface of the Sun and the Moon's shadow that corresponds to this special point. However, point P can be anywhere on the surface of the Sun. We thus wonder what would happen if we animate P, allowing it to move around the surface of the Sun. For each new instance of P, there is a new shadow left by the Moon on the Earth. Do the shadows overlap at all? Or will there be enough light rays to light all the shadows and make them less dark?

To find the overlapping of all those shadows as point P moves around, we thus enable the trace property for the two tangent lines from point P. By dragging P around or animating P, we can establish a pattern, if any, among all the shadows as shown in Figure 8. If the Moon is not too far away from the Earth, we can see that there is an area on the Earth that is shared by all the shadows, meaning that a person located in the area cannot see any light from the Sun, thus experiencing a total solar eclipse. As detailed in Figure 9, area T, called an umbra, is the area of total darkness during a total solar eclipse because there is no light ray from the Sun reaching that area. Around area T, the shadows are lit up by some sunlight, where a person can see part of the Sun, experiencing a partial solar eclipse. The area around T is thus called the penumbra. Having come to understand these basic ideas behind a total solar eclipse, students should be encouraged to pose more what-if questions [3] and use *GeoGebra* to look into the consequences. For example, what if the Moon is farther away from the Earth? What if the Moon is much bigger or smaller relative to the Earth and the Sun? By exploring the various possibilities, students will gradually come to appreciate this spectacular astronomical event, including its physics, mathematics, and culture.

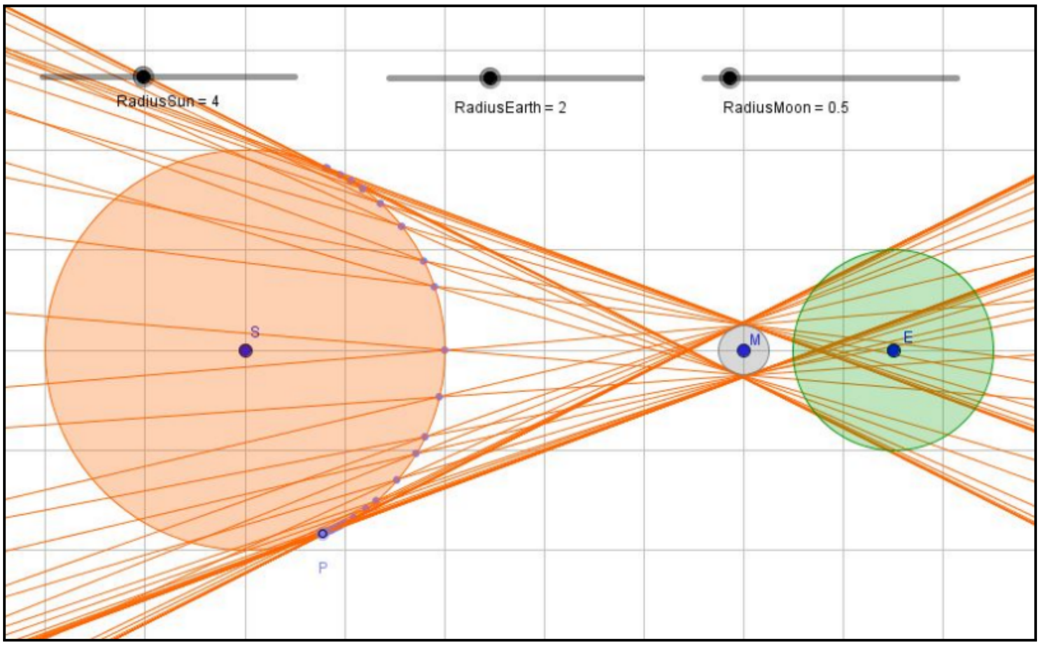

Figure 8. Using *GeoGebra* to find the overlapping of the Moon's shadows.

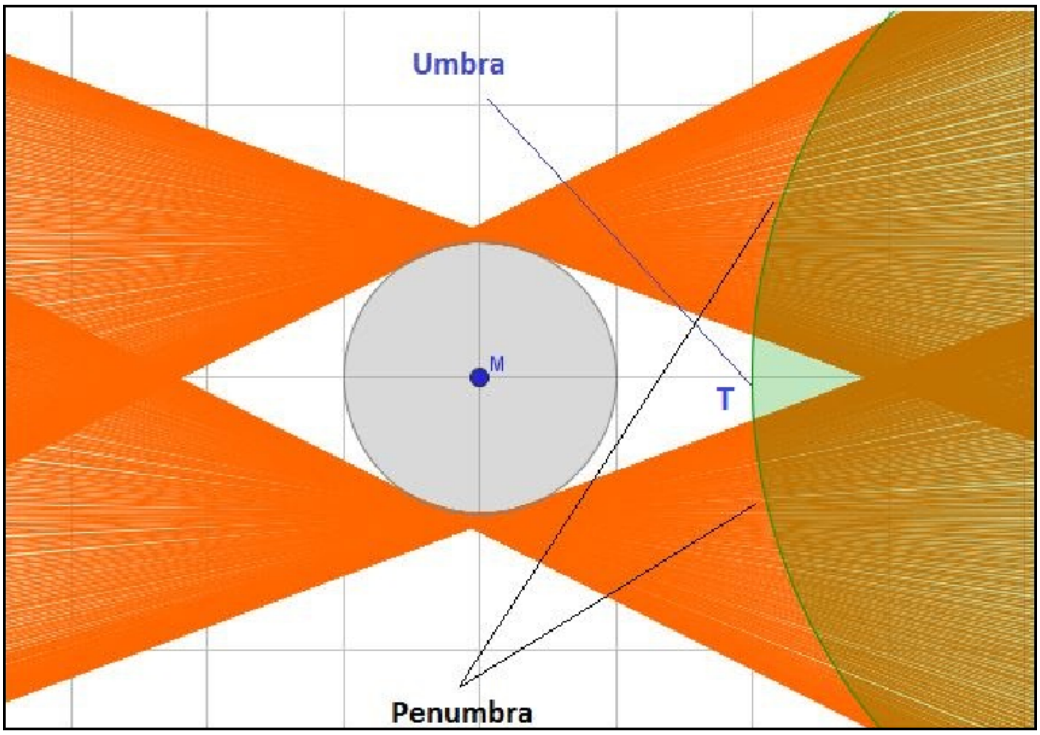

Figure 9. Using *GeoGebra* modeling to locate the umbra and the penumbra.

7 CONCLUSION

The total solar eclipse on August 21, 2017, stands as one of nature's enticing opportunities for educators of science, technology, engineering, and mathematics (STEM) to reach out to K-16 students and the public in terms of STEM awareness and education. Hundreds of thousands of students and other community members along the path of totality [5] will have an unforgettable chance to appreciate the power of science and mathematics when they come face to face with a total solar eclipse. In this context, *GeoGebra* allows us to build a reasonable model to make sense of the dynamics among the Sun, and Moon, and the Earth during a total solar eclipse. We used only a few technical features of *GeoGebra*, namely the circle tools, tangent lines, and tracing, to showcase the processes behind a total solar eclipse. In a classroom teaching experiment in spring 2016, we used the *GeoGebra* model with a group of 27 elementary teacher candidates with great success. Prior to the *GeoGebra* modeling, few teacher candidates could explain what is behind a total solar eclipse in terms of a physical or mathematical process. After the intervention, the majority of the teacher candidates indicated that they understand the basic processes behind a total solar eclipse and that the dynamic *GeoGebra* model played an important role in helping them see the why behind a total solar eclipse. They further felt that they could use *GeoGebra* to explain it to school children and other community members. The majority also expressed an interest to learn more about the upcoming solar eclipse through other resources.

Indeed, there is much more behind a total solar eclipse than what we discussed above. Our mathematical approach focuses on the basic structure and the process of the phenomenon, leaving out such details as the orbits of the Earth and the Moon and their tilting angles, which have significant consequences in the solar system. It is noteworthy that the *GeoGebra* model in Figure 6 can be easily modified (so that the Moon is out of alignment with the Sun and the Earth) to explore the effect of the Moon's orbit tilting on a solar eclipse. Further, the solar eclipse occurs in a 3D space, where both the umbra and penumbra are 3D cones. Our cross-sectional model does not represent the 3D nature of the phenomenon. These limitations should be communicated to the students and the public when the above *GeoGebra* model is used for explaining the total solar eclipse. If a 3D model is pedagogically preferred for a certain audience, the 2D model mentioned above can be translated into a 3D model within the *GeoGebra* 3D Panel, where 3D cones can be used instead of tangent lines to model light rays. However, to see the umbra of a total solar eclipse, we have to "look into" the cones, which means we need to construct a cross-sectional view of the 3D model (please refer to Figure 10 and the URL in the Appendices). In summary, all mathematical models, including *GeoGebra* models, have affordances and limitations and thus should be used for its specific purpose with the targeted audience. As shown in the article, *GeoGebra* provides a powerful open platform for STEM educators to introduce modeling into a school classroom as a pedagogical stance [4] and further help students develop a mathematical perspective on this astronomical spectacle and numerous similar real-world problems. Finally and most importantly, although it is perfectly safe to stare at a mathematical model on a computer screen, it is extremely dangerous to look at the Sun without proper protective glasses even for a second. Therefore, we should inform our students and the public to follow the safety procedures in their observation of the solar eclipse. For more information about the total solar eclipse, readers could visit `www.eclipse2017.org` or related NASA web resources [8].

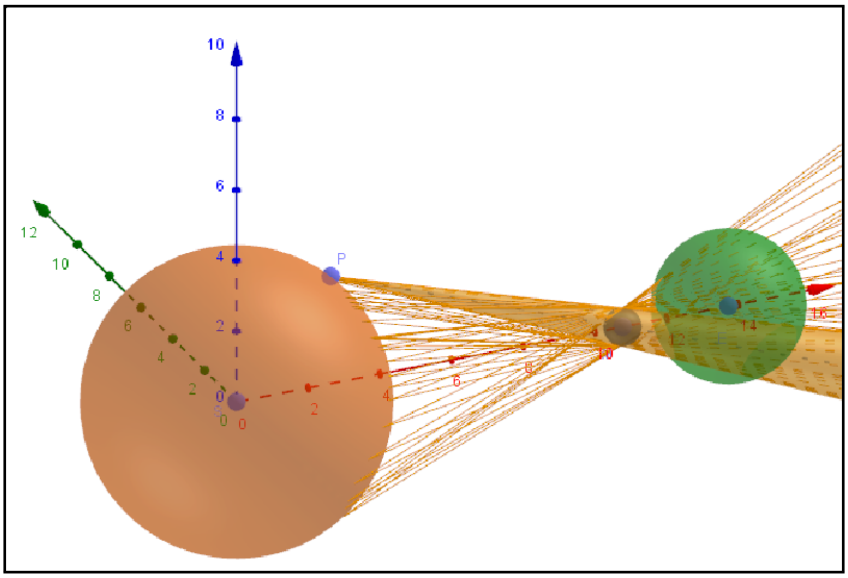

Figure 10. A *GeoGebra* 3D model for a total solar eclipse, where cone outlines are traced to show the presence of an umbra.

References

[1] Blair, W. P. (2001). *Some basic astronomy demonstrations for early elementary ages*. Available at http://www.pha.jhu.edu/~wpb/globe.html

[2] Blair, W. P. (2004). *Size scales in astronomy*. Available at http://www.pha.jhu.edu/~wpb/scale.html

[3] Brown, S. I., & Walter, M. I. (2005). *The art of problem posing* (3rd ed.). Mahwah, NJ: Lawrence Erlbaum Associates.

[4] Council of Chief State School Officers [CCSSO]. (2010). *Common Core State Standards for Mathematics*. Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers. Available at http://www.corestandards.org/math

[5] *Eclipse 2017*. Available at http://www.eclipse2017.org/2017/path_through_the_US.htm

[6] Espenak, F. (2009). *Solar eclipse for beginners*. Available at http://www.mreclipse.com/Special/SEprimer.html

[7] Espenak, F. (n.d.). *Periodicity of solar eclipses*. Available at http://eclipse.gsfc.nasa.gov/SEsaros/SEperiodicity.html

[8] National Aeronautics and Space Administration [NASA] (n.d.). *NASA eclipse website*. Available at http://eclipse.gsfc.nasa.gov/solar.html

[9] Schiffer, M. M., & Bowden, L. (1984). *The role of mathematics in science*. Washington, DC: Mathematical Association of America.

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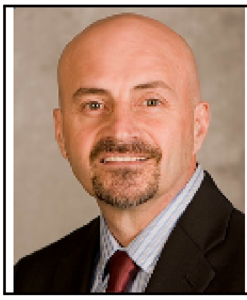

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APPENDICES

(1) A dynamic webpage for the *GeoGebra* model is available at `https://www.geogebra.org/m/Yw5xagAz`
(2) A dynamic webpage for the 3D *GeoGebra* model is available at `https://www.geogebra.org/m/KK6fAJpF`
(3) Outline of *GeoGebra* Commands for the Solar Eclipse Model

Step	**Object**	**Description**
1	Point S	Center of the Sun
2	Point M	Center of the Moon
3	Point E	Center of the Earth
4	Variable RadiusSun	Radius of the Sun with initial value of 4 units, visualize as a slider, change properties as necessary
5	Variable RadiusEarth	Radius of the Earth with initial value of 2 units, visualize as a slider, change properties as necessary
6	Variable RadiusMoon	Radius of the Moon with initial value of 0.5 units, visualize as a slider, change properties as necessary
7	Circle for the Sun	Circle with center S and radius RadiusSun
8	Circle for the Moon	Circle with center M and radius RadiusMoon
9	Circle for the Earth	Circle with center E and radius RadiusEarth
10	Point P on Circle for the Sun	Pick a free point P on the circle for the Sun
11	Tangent lines from P to the Circle for the Moon	Create two tangent lines from point P to the circle for the Moon
12	Enable trace	Enable Trace for the two tangent lines
13	Drag point P	Drag point P on the Sun to simulate a solar eclipse
14	Tweak the variables and the relative locations of the Sun, the Moon, and the Earth for open-ended exploration.	

Please note that a step-by-step demonstration can be initiated within the *GeoGebra* file using the Construction Protocol under View.