FOSTERING GROWTH IN TEACHER CONTENT KNOWLEDGE THROUGH THE CONSTRUCTION OF SKETCHES FOR STUDENTS

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Abstract: In this paper, the author investigates connections between teacher content knowledge and the construction of sketches through an examination of teacher-constructed applets. Specifically, the author analyzes the mathematics that teachers needed to know to design sketches in GeoGebra to support student understanding of a proportional reasoning problem aligned to 6th grade Common Core math standards.

Keywords: pedagogical content knowledge, teacher knowledge, problem solving, proportional thinking, modeling

Introduction

A good deal of research has explored the use of dynamic geometry software as a tool for strengthening student understanding of various secondary school topics [1, 3, 5]. Less common are studies that investigate connections between teacher content knowledge and the construction of sketches. What mathematics content do teachers learn as they create sketches for their students? How does this content compare to the content that teachers convey to students through such sketches? In the following discussion, we respond to these questions through an examination of teacher-constructed applets that support student work on Ty's Escalator Task [2]:
Ty took the escalator to the second floor. The escalator is 12 meters long, and he rode the escalator for 30 seconds. Which statements are true? Select all that apply.

1. He traveled 2 meters every 5 seconds.
2. Every 10 seconds he traveled 4 meters.
3. He traveled 2.5 meters per second.
4. He traveled 0.4 meters per second.
5. Every 25 seconds, he traveled 7 meters.

Note that the task requires students to draw various conclusions about Ty's speed - a concept that is well suited for visualization with dynamic mathematics software. Through the analysis of two teacher-constructed models, we explore how the construction of sketches intended to support student exploration may strengthen teacher understanding of proportions and various other topics within the school mathematics curriculum.

Although the teacher-constructed applets we explore in this paper provide nearly identical end-user experiences for students, the underlying mathematics that teachers used to create the applets are remarkably different. We posit that providing teachers with time to construct their own applets and share construction strategies with each other provides a novel way to foster content knowledge growth among teachers. Such explorations provide teachers with opportunities to see connections between various areas of the school mathematics curriculum that are too often overlooked.

Finished Sketch: End-User Experience

In a summer professional development experience, specifically a keynote presentation at the 2015 Midwest GeoGebra Conference, Ty's Escalator Task was shared with a group of approximately 100 secondary-level school teachers. In addition, teachers were shown a dynamic sketch that accompanied the problem. The sketch, constructed by the authors of this paper, is illustrated in Figure 1.1.

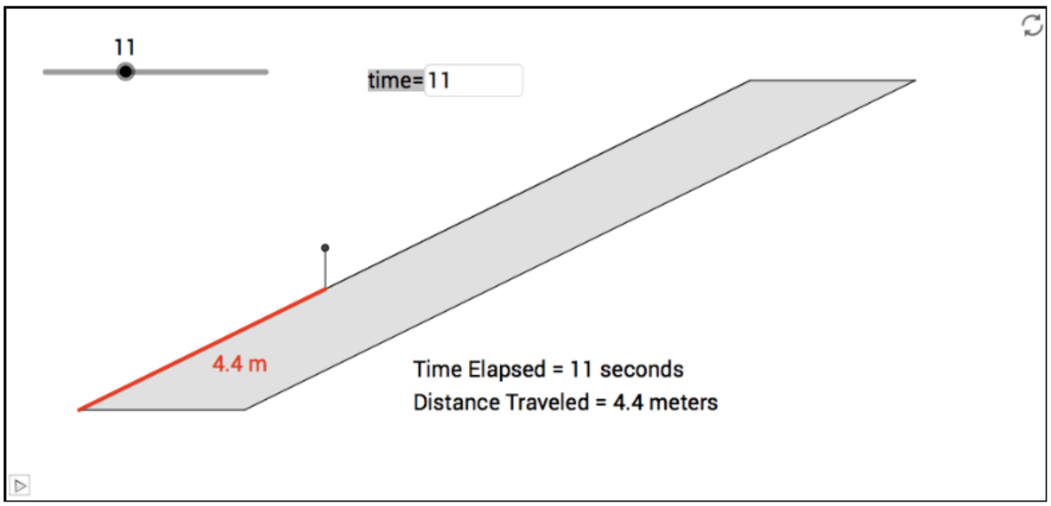

Figure 1.1 Sketch of Ty's Escalator Task (available on-line at http://tube.geogebra.org/m/1297645)

In small groups, teachers were asked to examine the task and the sketch, then brainstorm features of the applet they felt would help students. Ideas for enhancing the sketch were also solicited. Through audience participation, teachers noted the following positive features.

- The variable, `time`, is controlled by dragging the slider in the upper left corner of the sketch. Students may also enter a specific time in the input box to the immediate right of the slider.
- Specific values of `time` and `distance` are shown below the escalator. These values depend upon the value of time and are updated automatically as the slider is manipulated.
- The total distance that Ty has traveled up the escalator is highlighted in red as a line segment.
- The slider changes values of time in increments of 1 second. Pressing the play button in the bottom left corner of the sketch automates the movement of the slider (and, therefore, the movement of Ty).
- Including automation enables students to visualize the escalator movement in real-time, focusing full attention on number relationships in the sketch without the distraction of having to move the slider.

The following were offered as possible enhancements:

- Provide students with a means to analyze captured time and distance data within a graph or table. Students will likely benefit from seeing multiple representations of the same data.
- Provide the option for students to modify the length and speed of the escalator. This would encourage students to generalize their findings.

The underlying strategies used to construct the sketch in Figure 1.1 were not shared with teachers. Instead, teachers were encouraged to build their own versions of the sketch using GeoGebra. We encouraged teachers to enhance the design of the sketch in any way they saw fit. At the end of the day-long meeting during an optional debriefing session, participants shared their sketches.

In the sections that follow, we explore the design approaches of two teachers. These were selected because of their relative simplicity and because the underlying mathematics used to construct the two applets was quite different. Discussions of these (and other) strategies during the debriefing provided teachers with opportunities to make connections between various areas of mathematics as they learned more about the GeoGebra software.

Construction Method 1: Coordinates and Component Vectors

The first teacher modeled the escalator ride by considering Ty's horizontal and vertical movement separately within the x-y coordinate plane. Figures 1.2-1.6 provide step-by-step screenshots of such a construction. First, the teacher constructed line segment $\overline{AB}$, with A fixed at the origin, as shown in Figure 1.2. The placement of B was arbitrary.

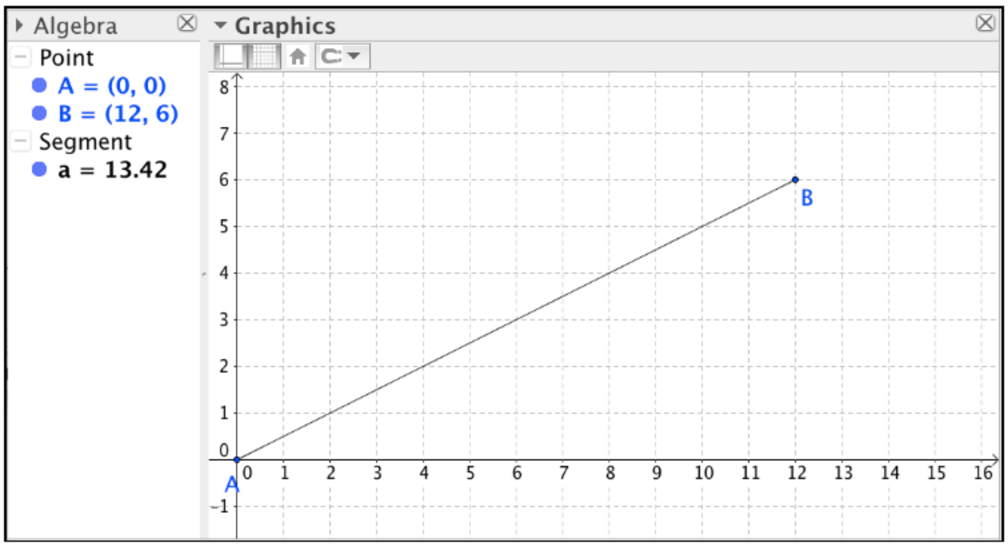

Figure 1.2 Segment AB with A fixed at the origin.

Next, the teacher constructed a slider with values ranging from 0 to 30 seconds, incremented by 1 unit. Slider settings are depicted in Figure 1.3.

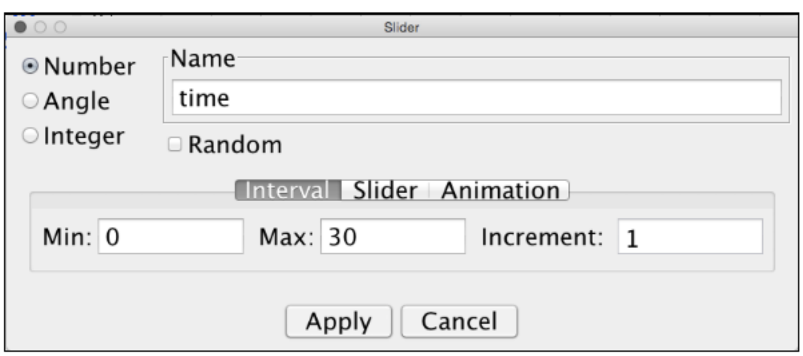

Figure 1.3 Settings for time slider.

With `time` defined, the teacher defined Ty's horizontal and vertical movement as separate variables – `hdist` and `vdist`, respectively.

Next, the teacher defined Ty's location on the x-y plane, P, in terms of these variables. Since Ty's starting point is placed at the origin, point A, the x-coordinate of point B, `x(B)`, describes the total horizontal distance Ty travels. Likewise, `y(B)` describes the entire distance Ty travels in the vertical direction. Multiplying horizontal and vertical components of distance traveled by the fraction of the total time elapsed, `time/30`, yields Ty's horizontal and vertical location at the current time. Hence, `P=(hdist,vdist)` describes Ty's current location.

These steps, illustrated in Figure 1.4, consider Ty's movement as component vectors.

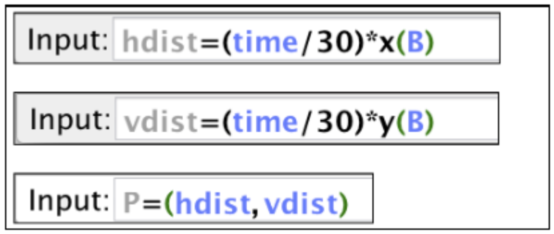

Figure 1.4 Definition of `hdist`, `vdist`, and point `P` (Ty's position on the escalator).

Interestingly, the teacher who shared this construction was surprised when others suggested her work implemented a vectors-based approach. As a geometry teacher, she did not teach her students about vectors and had not worked with vectors formally "since college." The teacher commented that she "hadn't thought of it that way" and that her motivation for using x- and y- components separately stemmed from a desire to break down the construction into "bite-sized" steps. Clearly, through our discussion of this sketch, a number of teachers strengthened their understanding of vectors and connected them to the content typically found in entry-level courses.

Figure 1.5 illustrates the teacher's sketch with the aforementioned variables defined. Figure 1.6 shows a completed sketch with gridlines and the Algebra View hidden.

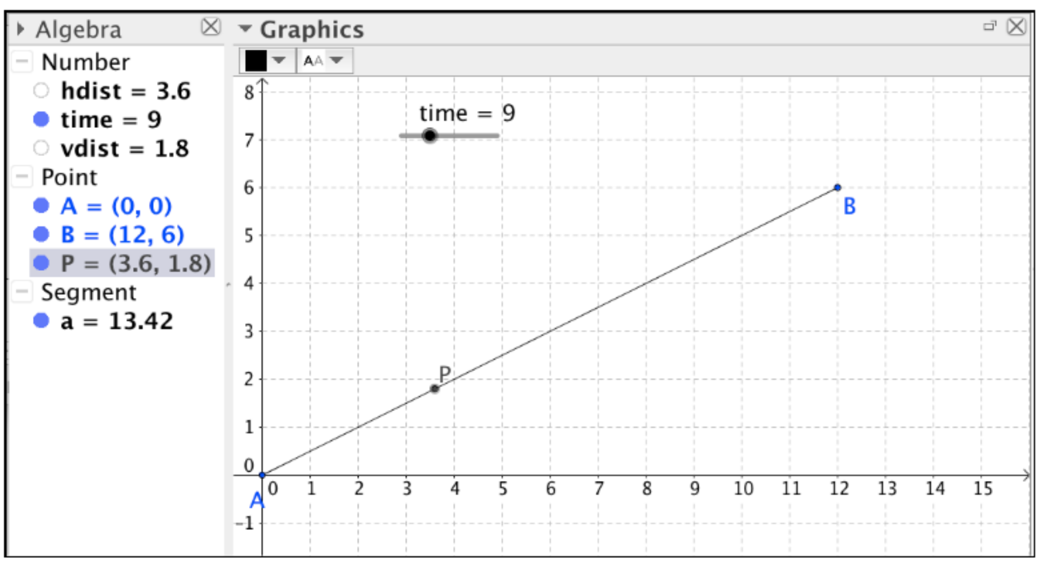

Figure 1.5 Sketch including Ty's position on the escalator with respect to time.

Construction Method 2: Similar Circles

While the component vector approach described in the previous section has a decidedly algebraic flavor, geometry yields an alternative construction that is arguably more elegant and more accessible to students in entry-level courses. Figures 1.7-1.11 provide step-by-step screenshots of a teacher-generated approach offered by a second teacher. First, using the "Circle with Center and Radius" tool, the teacher constructed a circle with fixed radius of 12 units centered at point A. The results of this are shown in Figure 1.7.

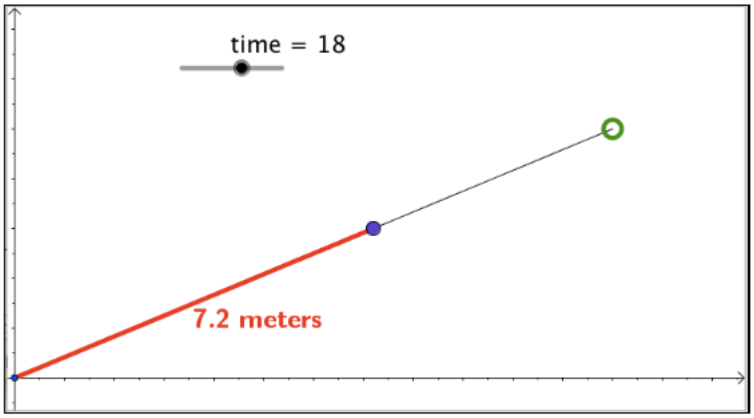

Figure 1.6 Completed sketch implementing a component vector approach.

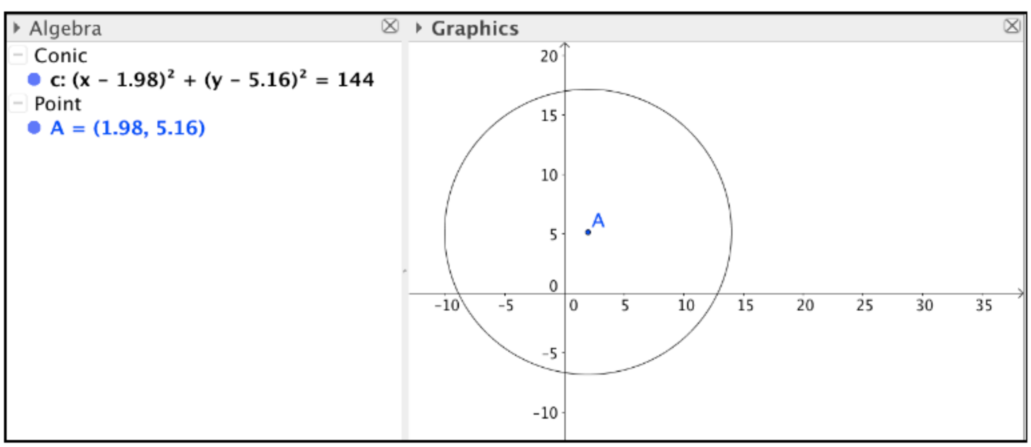

Figure 1.7 Construction of a circle with fixed radius of 12 units.

Next, the teacher constructed a point B on the circle followed by radius AB. This radius, shown in Figure 1.8, represents the entire length of the escalator.

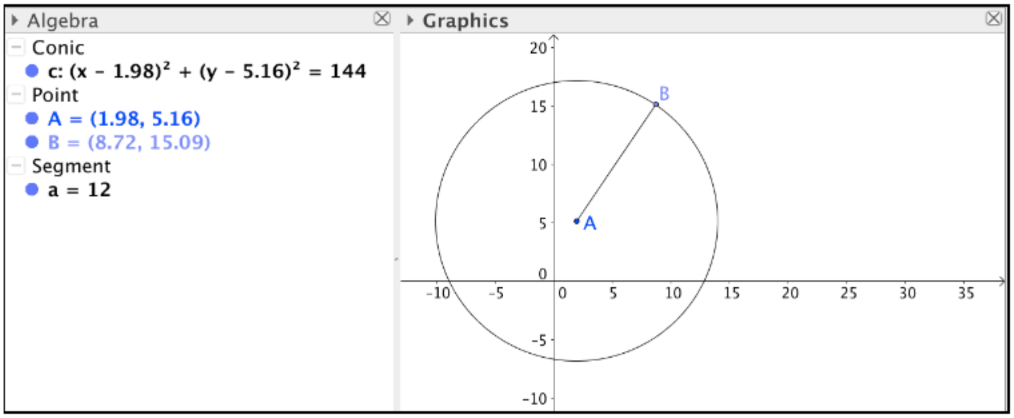

Figure 1.8 Segment $\overline{AB}$ representing the escalator in Ty's Escalator Task.

Sitting at a table in a common meeting area, the teacher explained to us that she used the radius of a scaled circle to represent the distance Ty traveled up the escalator at time, t. After creating slider t, she constructed a second circle centered at A with scaled radius. She defined the radius length as shown in Figure 1.9. Note that a is the length of segment AB while t/30 is the fraction of the total time that has elapsed during Ty's trip.

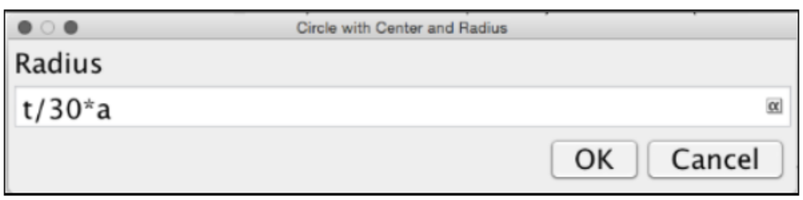

Figure 1.9 Constructing a second circle with scaled radius.

The results of this construction are illustrated in Figure 1.10. The point at which the scaled circle intersects the original radius AB represents Ty's location on the escalator at time, t.

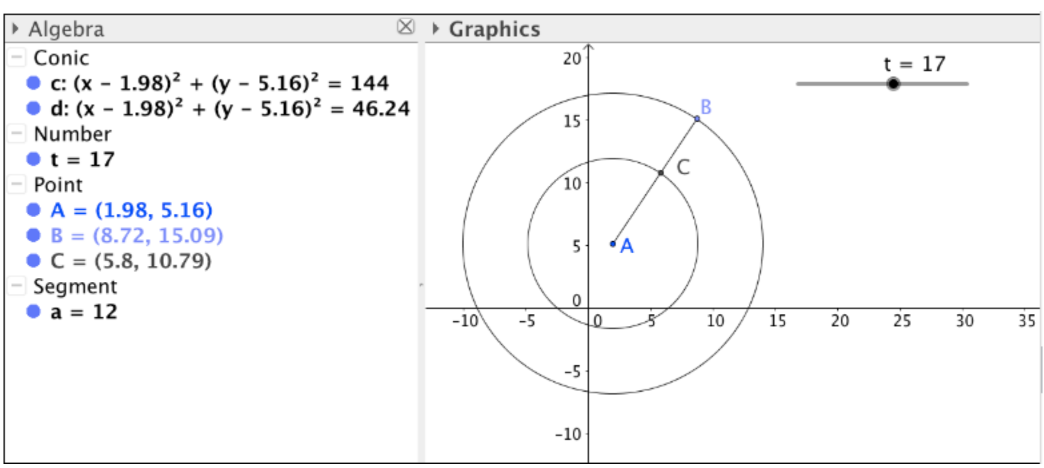

Figure 1.10 Ty's location on the escalator as represented by the intersection of a scaled circle and radius AB.

Figure 1.11 shows a completed sketch with the grid, axes, circles, and Algebra View hidden.

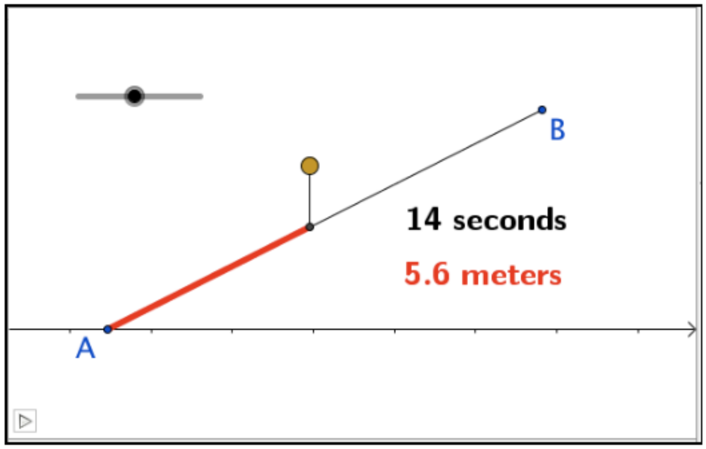

Figure 1.11 Completed sketch implementing similar circles approach.

In this second approach, the teacher essentially constructed a scaled circle using a size change transformation with scale factor t/30. This observation was an epiphany for a handful of our participants, particularly those who did not teach geometry and had a tendency to think of proportional situations numerically rather than geometrically.

Observations and Implications

Although the construction approaches illustrated in the preceding discussion are markedly different—one uses the underlying idea of component vectors; the other uses scaled circles—the resulting sketches are remarkably similar. For instance, both final sketches depict the escalator as a line segment. Both use a slider to control time. Both display time elapsed and distance traveled as dynamic text. When teachers shared their construction methods during the debriefing session, they were surprised by the variety of construction ideas that their colleagues implemented. As one teacher commented, "I had no idea there were so many ways to do the same thing with the software."

In general, four basic approaches were used by teachers to construct a model of Ty's Escalator Task, namely (a) geometric scaling, (b) numerical scaling, (c) function and equation scaling, and (d) tabular scaling (using GeoGebra's spreadsheet features, these teachers diverged somewhat from the original task of duplicating the escalator, focusing on determining escalator speeds instead). Sharing various construction strategies appeared to deepen teacher understanding of ratios and proportionality, particularly for those who tended to think of proportional situations either as numerical or as geometric, but not both. In addition to the usual task of finding ratios and their equivalents, which can be accomplished numerically, the modeling activity engaged teachers in more interesting discoveries beyond calculating ratios. As teachers shared strategies, they connected content from throughout the K-12 curriculum, including vector algebra, transformational geometry, and advanced algebra.

It is interesting to note that the content that students explore with the finished sketches focuses primarily on rates. The finished sketches in Figures 1.6 and 1.11 do not immediately suggest radii of concentric circles or vectors, yet this content was instrumental in a number of teachers' construction strategies. This suggests that designers may learn mathematics content that is above and beyond that which is conveyed to students through finished sketches. This has implications for ways in which teachers use dynamic sketches with their students. For instance, rather than engaging students with ready-made sketches in all cases, teachers should consider providing their students with opportunities to construct their own sketches. Doing so encourages students to connect content that they are currently learning to content from other courses in meaningful and creative contexts.

REFERENCES

1. Hölzl, R. (2001). Using dynamic geometry software to add contrast to geometric situations: a case study. *International Journal of Computers for Mathematical Learning, 6*(1), 63-86.

2. Illustrative Mathematics [website] (2015). 6.RP The Escalator Assessment Variation Available on-line at https://www.illustrativemathematics.org/content-standards/tasks/1181
3. Jones, K. (2000). Providing a foundation for deductive reasoning: Students' interpretations when using dynamic geometry software and their evolving mathematical explanations. *Educational studies in mathematics, 44*(1), 55-85.
4. Koehler, M. J., & Mishra, P. (2008). Introducing tpck. *Handbook of technological pedagogical content knowledge (TPCK) for educators*, 3-29.
5. Marrades, R., & Gutiérrez, A. (2000). Proofs produced by secondary school students learning geometry in a dynamic computer environment. *Educational studies in mathematics, 44*(1-2), 87-125.