EUCLID, THE GAME FOR VIRTUAL MATHEMATICS TEAMS

Joanne Caniglia

Kent State University

Abstract

The author discusses pedagogical benefits of a web-based teaching resource, Euclid, the Game, to explore various constructions from Euclid's Elements. Euclid, the Game *can be used with individual students or as a collaborative tool. When used within the Virtual Math Teams with GeoGebra (VMTwG), students observe characteristics of dynamic math figures, question their classmates' work, and discuss mathematics with others [4].*

Keywords: Euclid's Elements, geometric construction, collaborative mathematics

1 INTRODUCTION

This article presents a web-based teaching resource, *Euclid, the Game*, [3]. The game uses GeoGebra's [1] unique capabilities to engage students in constructions found in Euclid's Elements. *Euclid, the Game* can be used as with individual students or as a collaborative tool. When used within the Virtual Math Teams with GeoGebra (VMTwG), students observe characteristics of dynamic math figures, question their classmates' work, and discuss mathematics with others [4]. The VMTwG environment provides a virtual "lobby" for selecting mathematical activities, chat rooms, and a wiki for sharing ideas with other groups [4]. One of the types of tabs available in VMT chat rooms allows students to access a multi-user version of GeoGebra.

2 THE GAME

The game, available at `http://euclidthegame.com`, was created with Java in GeoGebra by Kasper Peulen [3]. Its premise is intuitive. Players are presented a series of constructions. The first level asks players to construct an equilateral triangle from a given line segment within an online GeoGebra sketch. Individuals are limited to using basic tools of geometry: points of intersection and on an object, segments, rays, and circles. Upon completing a level, results from previous levels may be used as additional tools (see Figure 1). Soon game play gets more difficult, and students are asked to trisect a segment or perform tasks such as the following: "Construct two new circles, given a circle of radius AB, where each pair of the three circles is tangent."

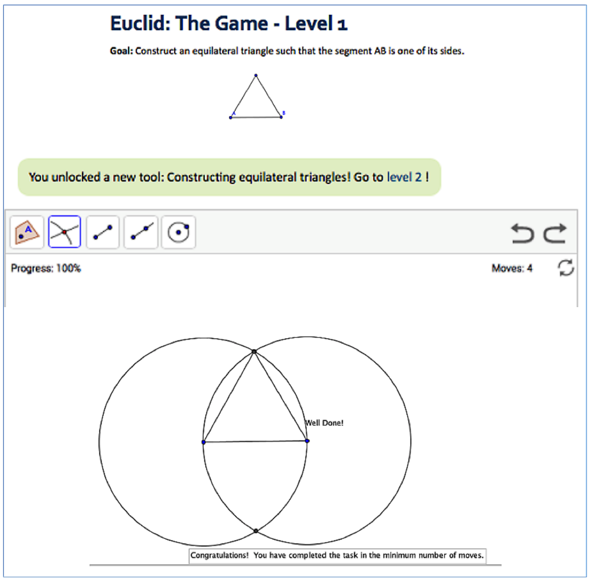

Figure 1. Completion of Level 1.

3 Design

Euclid, the Game observes design principles that encourage interest and lead to reasoning and development of logic. Principles include the following.

1. The student is given a platform (GeoGebra) to explore and manipulate. Students use computer tools to implement straightedge and compass constructions that Euclid describes in *Elements*. The constructions they create in *Euclid, the Game* are dynamic. Students move objects around, changing their measurements, but maintain the dependencies that they design into their constructions.

2. The variety of levels of *Euclid, the Game* provides a rich environment for exploration and, thus, is suitable for students from widely varying mathematical backgrounds and abilities. That said, students with stronger backgrounds in deductive reasoning perform at higher levels in the game. In addition, there are many possible "correct" constructions.

3. Feedback is direct, but not immediate, allowing students time to consider other potential solutions. Yet, when a player has completed part of the solution, there is a congratulatory message of "Well Done!" Scoring is based on the number of moves or steps within the construction. Finding points of intersection is not considered a move.

4. The directions provided in *Euclid, the Game* are clear. Each construction is displayed on the top of the next level's webpage along with the GeoGebra tools that are allowed in that particular level.

The goal of *Euclid, the Game* is to provide students with a rich, geometrically sound environment as well as the motivation to complete each level in the most efficient manner.

4 Euclid's *Elements* and the Game

Players are provided with two types of tools: primitive and derived. Primitives are pre-existing tools within the system. Derived tools are the result of student constructions. Among the GeoGebra primitive tools are line segments, rays, and circles (point-radius). For instance, in Level 1, the student completes four moves using primitive tools. Specifically, students use two circles and two line segments to create an equilateral triangle. The equilaterial triangle is a derived tool which can be used in subsequent levels of the game. In Level 4, for example, students construct a compass, which is extremely useful in later constructions. After Level 2, two different medals are available for a given level. When players accomplish a given task in the minimum number of moves, they receive a gold medal. If a level is completed correctly, but not with the minimum number of moves, a silver medal is awarded. Each medal includes a number inside indicating the number of moves the student needed to complete the exercise. Top scores are recorded. This is suggested in Figure 2.

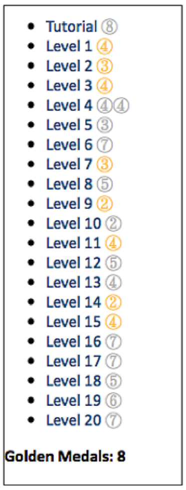

Figure 2. Score on the left side of the Euclid the Game screen.

5 Historical Connections

A comparison of Euclid's *Elements* and the 20+ levels of *Euclid, the Game* demonstrates how useful this type of technology is, not only for teaching constructions, but also when introducing the history of mathematics to students (Table 1). For example, after students attempt each level and succeed, they compare their results with those found in the *Elements*.

6 Virtual Math Teams with GeoGebra (VMTwG)

The VMTwG program has tabs and tools to help groups to explore mathematics collaboratively. After logging in, the first screen that students see is the VMT Lobby, as shown in Figure 3.

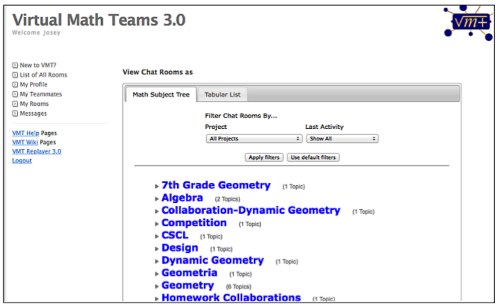

Figure 3. Lobby View of VMT (http://vmt.mathforum.org/VMTLobby).

Euclid's Elements [2]	Euclid the Game
Book I. Proposition I: On a given finite straight line to construct an equilateral triangle.	Level 1. Construct an equilateral triangle such that the given segment is one of its sides.
Book I. Proposition 10: To bisect a given finite straight line.	Level 2: Construct the midpoint of a given line segment.
Book I. Proposition 9: To bisect a given rectilineal angle.	Level 3: Construct a line (segment) that bisects the given angle.
Book I. Proposition 11: To draw a straight line at right angles to a given straight line from a given point on it.	Level 4: Construct a line (segment) that goes through a point on the line segment and that is perpendicular to the given line segment.
Book 1. Proposition 12. To a given infinite straight line, from a given point which is not on it, to draw a perpendicular straight line.	Level 5: Construct a line (segment) perpendicular to the given line going through a point not on it.
Book I. Proposition 31: To draw a straight line through a given point parallel to a given straight line.	Level 6: Construct a parallel line through point not on the line.
Book I. Proposition 2: (It is possible) to place at a given point (as a extremity) a straight line equal to a given straight line.	Level 7: Construct a line segment with the same length and same direction as the given segment.
Book 1. Proposition 3: Given two unequal straight lines, to cut off from the greater a straight line equal to the less.	Level 8: Construct a line segment with the same length and same direction as line segment but with a different vertex.
Book III. Proposition 17: Through a given point outside a given circle, construct a tangent to the circle.	Level 9: Construct a circle with radius equal to a given line segment and center not on the segment..
Book I. Proposition 2: (It is possible) to place at a given point (as an extremity) a straight line equal to a given straight line.	Level 10: Construct a new point on another line segment such that the new segment has the same length as the original.
Book I. Proposition 22: Out of three straight lines, which are equal to three given straight lines, (it is possible) to construct a triangle: thus it is necessary that two of the straight lines taken together in any manner should be greater than the remaining one.	Level 11: Construct a triangle whose sides have the same length as the given segments.
Book 1. Proposition 23. On a given straight line and at a given point on it, to construct a rectilinear angle equal to a given rectilinear angle.	Level 12: Construct an angle that is equal to a given angle.
Book III. Proposition 1. To find the center of a given circle.	Level 13: Find the center of the circle.
Book III. Proposition 17. From a given point to draw a straight line touching a given circle.	Level 14: Construct a line (segment) at a given point tangent to the circle. A tangent line to a circle is a line that only touches the circle at one point.
Book III. Proposition 4. In a given triangle to inscribe a circle.	Level 15: Construct the incircle of a triangle. An incircle is a circle fully contained in a triangle that is tangent to all three sides.
Book IV. Proposition 5 About a given triangle to circumscribe a circle.	Level 16: Construct the circumcircle of a triangle. A circumcircle is a circle that passes through all three points of a triangle
Book III. Proposition 17: Through a given point outside a given circle, construct a tangent to the circle. *	Level 17: Given a line, a line segment, and a point. Construct a circle with center that cuts off a line segment on the given line that has the same length as the given segment.
Book III. Proposition 17: Through a given point outside a given circle, construct a tangent to the circle & Proposition 32.	Level 18: Given a point, a line, and a point (on the line). Construct a circle that passes through the point and is tangent to the line at the point.
Book III. Proposition 18. If some straight-line touches a circle, and some (other) straight-line is joined from the center of the circle to the point of contact, then the straight-line so joined will be perpendicular to the tangent.	Level 19: Given a circle, with radius a fixed radius construct two new circles of radius same radius where one of the three circles must also touch a tangent point.
Book VI. Proposition 9. To cut off a prescribed part from a given straight line.	Level 20: Construct two points, such that the segment is cut into three equal pieces

Table 1: Comparison of Euclid, the Game and Euclid's Elements

VMTwG encourages collaborative learning through chats (in text form) about dynamic geometry activities (Stahl, 2013). The tool provides opportunities to discuss mathematics in small and large groups. This section will outline the steps to create a new chat room so that students can implement an adaptation of *Euclid the Game*.
Upon entering the Lobby, click on the link "My Rooms" on the left side. Select the tabs "Create New Room."

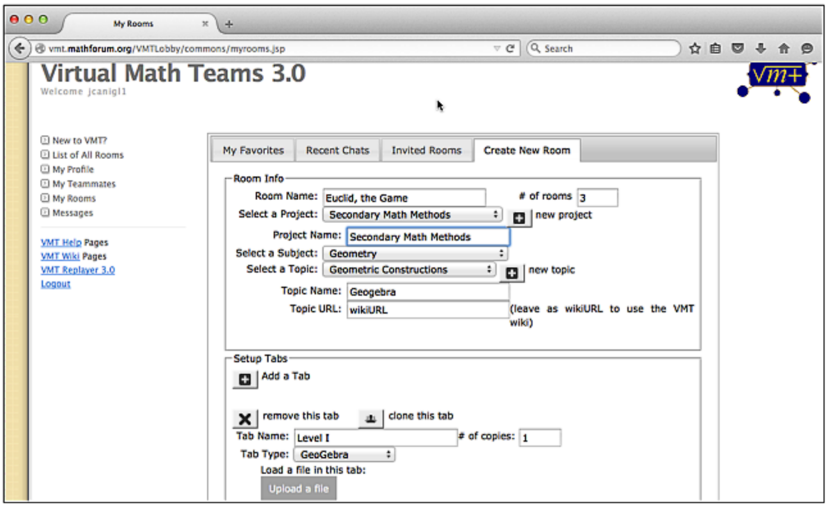

Figure 4. Interface to create a new chat room.

As Figure 4 suggests, the chat room interface is hierarchical. For instance, subjects exist within projects, topics exist within subjects, and rooms exist within topics. One can create multiple rooms at the same time for each team. Students must select from one of the pre-existing subjects. Subjects cannot be created or modified by students.

7 Combining the Two Environments

Because the levels from *Euclid, the Game* are not uploadable within the VMTwG environment in their entirety without the loss of scoring or text descriptions, the creators of VMT suggest alternatives for combining VMT and *Euclid, the Game* (email communication with Anthony Mantoan, lead programmer of VMT and Gerry Stahl, creator of VMT). These include the following:

1. Modify the tools of GeoGebra to align with *Euclid the Game*. With these tools in place, one can add script and figures similar to the levels of *Euclid, the Game*. Upon saving a GeoGebra file (.ggb), one can then upload it to the VMTwG room. Unfortunately, the VMTwG cannot keep score as *Euclid, the Game*, but the chat room can record the number of moves a student has played.

2. Gerry Stahl has created an interesting set of challenges that he has provided on GeogebraTube `http://tube.geogebra.org/student/b154045`. Stahl intended these to be used within the VMT environment with teams working on the sequence collaboratively while gaining significant knowledge of dependencies as an introduction to proof.

3. Anthony Mantoan suggests to log into VMT and change the project to vmtTest, go to subject, *Room Test*; topic, *Testing*. There are a few Euclid game rooms there.

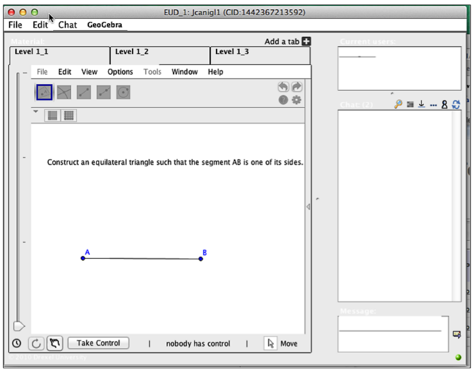

Figure 5. Customized GeoGebra tools.

7.1 Notes

Gerry Stahl is a researcher and professor emeritus of information science at Drexel University. His current research (from 2003 to the present) focuses on the Virtual Math Teams (VMT) project Drexel University's College of Computing and Informatics, the Math Forum and Rutgers-Newark. This project is extensively documented in his book, *Studying Virtual Math Teams*. His research team uses chat interaction analysis to explore what takes place in online discussion of math by students. His book, *Translating Euclid*, discusses the redesign of geometry education in terms of cognitive history, contemporary philosophy, school mathematics, software technology, collaborative learning, design-based research, CSCL theory, developmental pedagogy and scaffolded practice. For a complete list of his publications please see `http://gerrystahl.net/`.

References

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[2] Densmore, D. (2002). *Euclid's Elements: all thirteen books complete in one volume: the Thomas L. Heath translation*. Green Lion Pr.

[3] Peulen, K. (2015). *Euclid, The Game* [Computer software]. Available on-line at `http://euclidthegame.com`.

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Joanne Caniglia (jcanigl1@kent.edu)teaches in the School of Teaching, Learning, and Curriculum Studies at Kent State University in Kent, Ohio. Her research activities include creating meaningful mathematical tasks to assist secondary and special education students. She is a PI and Co-PI for many National Science Foundation and Board of Regents initiatives.