ARTISTIC SYMMETRY: FINDING RICHNESS IN TATTOO DESIGN USING GEOGEBRA TO EXPLORE SYMMETRY AND CREATE SYMMETRIC DESIGNS FOR A HIGH SCHOOL GEOMETRY COURSE

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Abstract

In this article, we describe an interdisciplinary tattoo project that provides students with opportunities to connect art, symmetry, and transformational geometry in engaging, exploratory, and personally meaningful ways. Students us the GeoGebra pen tool and various transformational geometry tools as they create various tattoo designs.

Keywords: geometry, transformation, connections, Common Core

1 THE CHALLENGE

During a graduate level advanced methods course, I was challenged to create a rich task using GeoGebra that would address the needs of three different levels of students. What follows is a description of my response to this challenge - namely, the creation of an interdisciplinary tattoo project that provides students with opportunities to connect art, symmetry, and transformational geometry in engaging, exploratory, and personally meaningful ways. Figure 1 illustrates several sample tattoo designs that students can construct within GeoGebra as part of the project. I provide details regarding the construction of these images in subsequent sections of this paper.

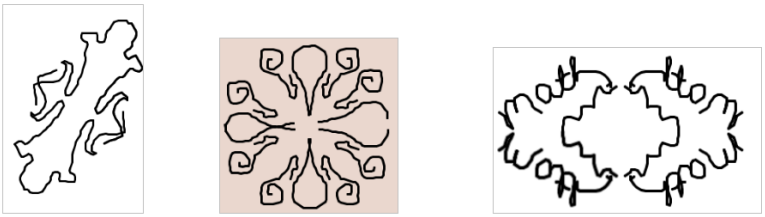

Figure 1. Three symmetric tattoo designs constructed using GeoGebra drawing and transformational geometry tools.

2 WHAT'S A RICH TASK?

My answer to this question has evolved over the past couple of years while working on my master's degree. To begin, a rich task is a problem posed to students which may be solved using several

different strategies by students at varying levels. Such tasks are often open-ended, typically having many different solutions. Rich tasks may or may not be real-world applications. Ideally, the process of solving a rich task is primarily a student-led process. Teachers serve as facilitators of learning, prompting students only when necessary. In the end, the process not only helps students comprehend the mathematics underlying the task, but also aids in developing and honing student problem-solving skills.

3 WHY RICH TASKS?

3.1 Common Core Standards

First and foremost, rich tasks are alluded to in the Common Core State Standards for Mathematics (CCSSM). The creation of CCSSM was prompted, in large part, by the desire of mathematics educators, policy makers, and classroom teachers to address criticisms that the current mathematics curriculum in the United States was a "mile wide and an inch deep." The new standards have been designed to encourage students to delve deeper into fewer concepts. Since a major outcome of the use of rich tasks is the deepening of student comprehension, their use supports the standards' primary objectives. Moreover, the Standards for Mathematical Practices are essentially a list of strategies required to solve rich tasks. According to the authors of the CCSSM, mathematical practices "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 6)

3.2 Differentiation

Furthermore, rich tasks are well-suited for differentiation. Since the teacher gives limited prompting, students use strategies that make sense to them to solve problems. Solution strategies may range from very basic methods, such as counting, data observation, or even trial and error, to more sophisticated approaches such as solving equations, constructing formal proofs or calculus-based approaches.

4 WHY TATTOOS?

In order to engage my students, I decided to select a topic which would likely capture their interest. My thoughts kept drifting toward tattoos. Often times, my students show off their new "tat" or share with me designs they've created in anticipation of getting "inked." This gave me a fantastic idea! I decided to use tattoo design to teach my geometry students about symmetry.

Figure 2. Tattoos are a popular art form among today's students.

5 THE SKETCHES

I created three different sketches for students to use. Two include imported images of Roman mosaic and Escher designs. The third is a blank 'Design Creator' that students may use to create or analyze the symmetry of their designs.

5.1 Roman Mosaic Sketch

Knowing my students' background, I followed the advice of Martin(2011), "If your students have not recently worked with rotational symmetry, you might begin with a review of the topic. An online applet that demonstrates rotational symmetry might be useful" (p. 2). Thus, the project begins with a student investigation of the symmetry of a Roman mosaic design. Students are initially provided with the GeoGebra sketch highlighted in Figure 3.

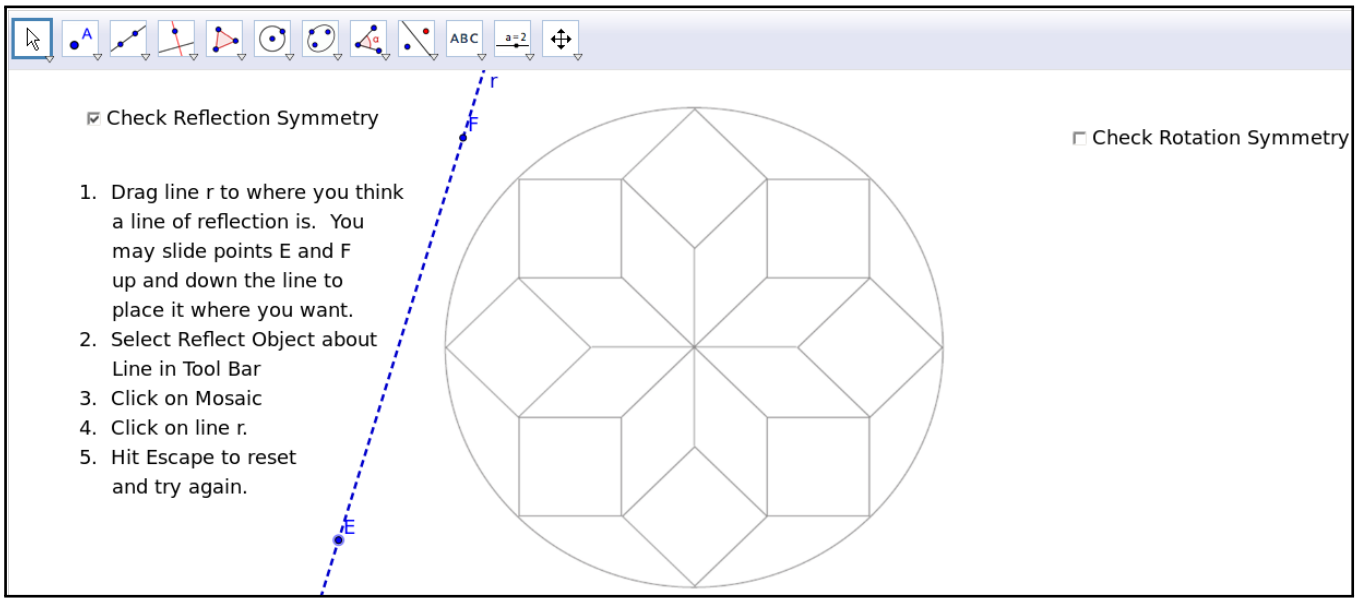

Figure 3. Roman Mosaic Sketch
Available on-line at `http://bit.ly/romanmosaic2`

The sketch in Figure 3 allows students to explore different centers of rotation and reflection lines while making predictions about the symmetry of the design.

5.2 Escher Angels and Demons Sketch

Students explore hypotheses regarding symmetry of pre-existing artwork a second time with an M.C. Escher print, "Angels and Demons." In this piece, the symmetry is not as easily detected. Functionality of the sketch is highlighted in Figures 4 and 5.

5.3 Design Creator Sketch

The task culminates with students designing their own tattoos with specific symmetric constraints. Images of tattoos are imported into an empty sketch, the "Design Creator" sketch. The sketch is essentially identical to earlier sketches with one exception - namely, all pre-selected images are removed. This is illustrated in Figure 6.

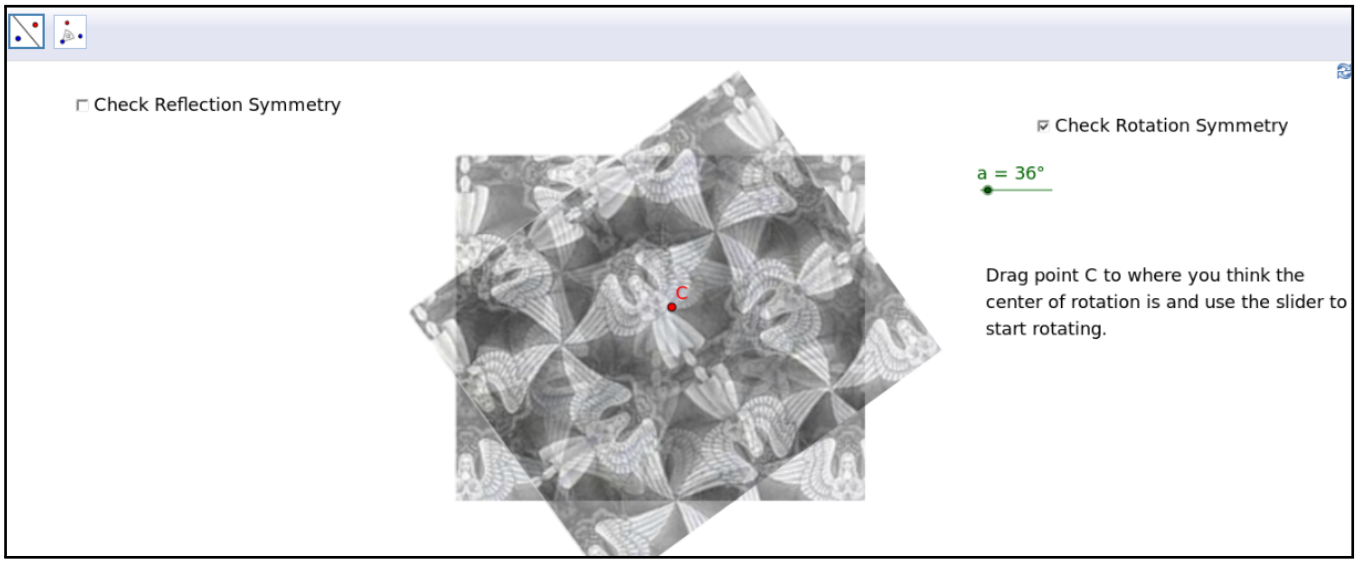

Figure 4. Students can click and drag the red point, C, to predict the location of the center of rotation. Available on-line at `http://bit.ly/escherangelsdemons`

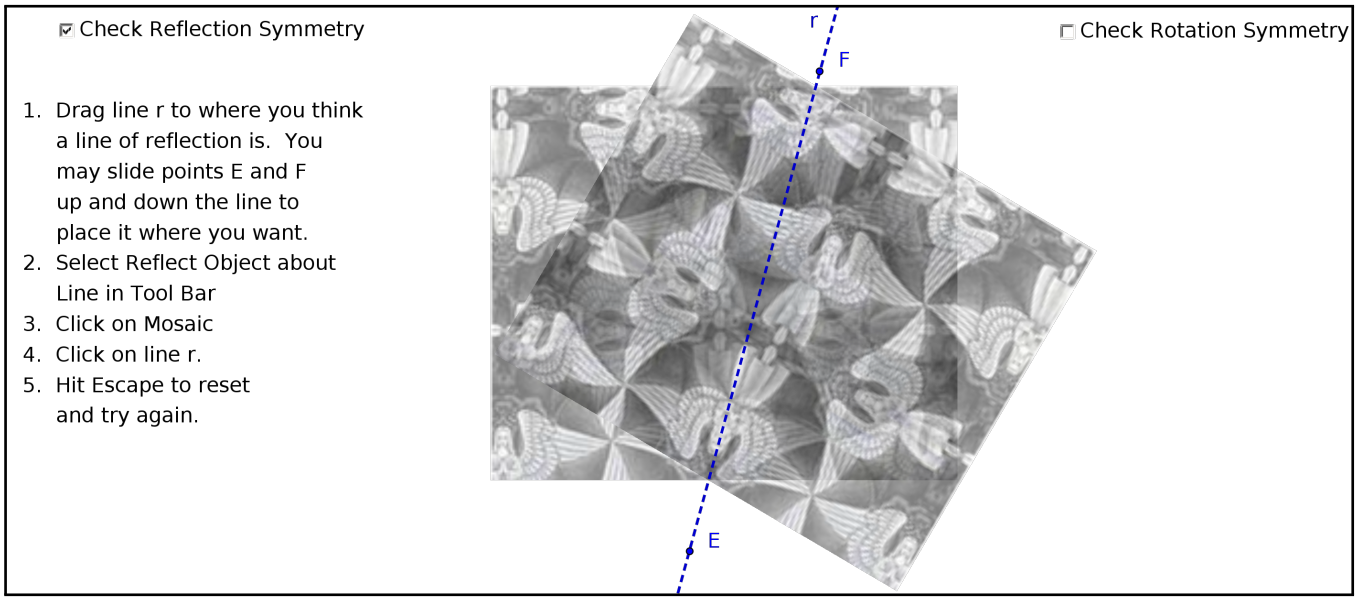

Figure 5. Students can position the reflection line at locations of their choosing to test predictions regarding the location of possible line(s) of reflection.

Students can use the sketch highlighted in Figure 6 or a blank one to design their tattoo. If necessary, students can scan their own image, insert it into the sketch, and test for symmetry.

During peer-review meetings in class, my colleagues indicated that it might be advantageous to allow students to simply create a design using a blank sketch rather than using the "Design Creator" sketch. They felt that the checkboxes and text took up too much room. As an alternative, students can easily create a tattoo design using a blank sketch and then import the image into the "Design Creator" when done to check for symmetry.

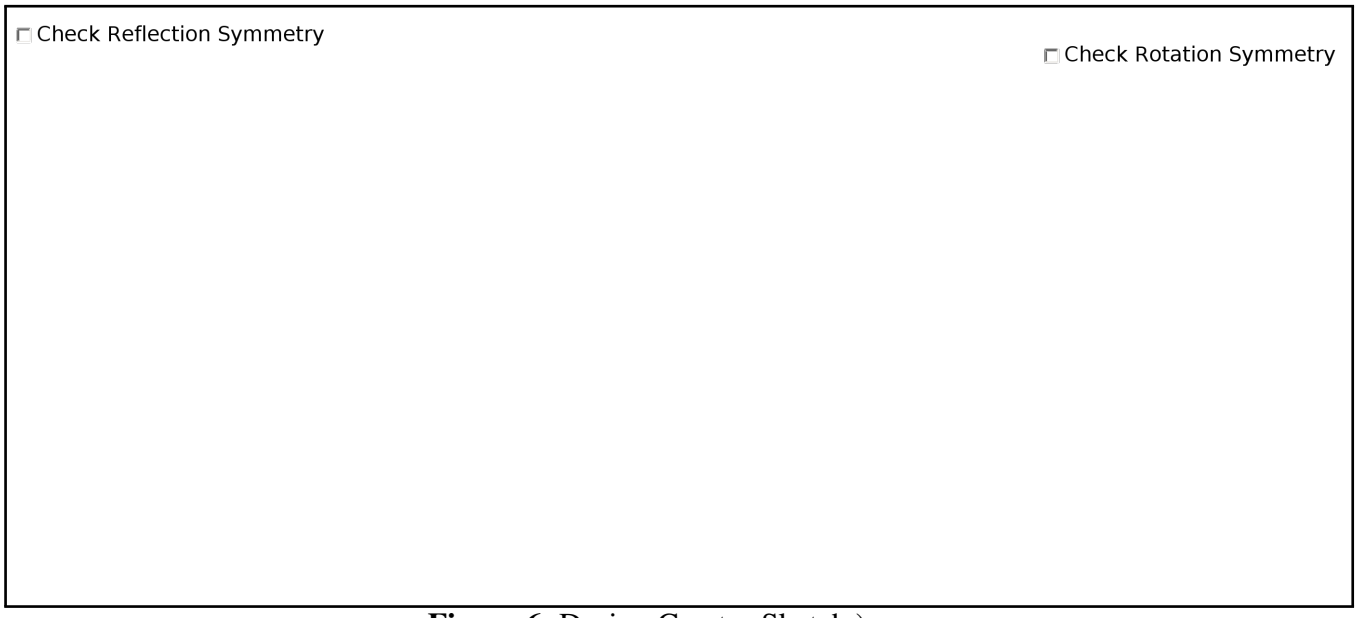

Figure 6. Design Creator Sketch.)
Available on-line at `http://bit.ly/designcreator`

6 DIFFERENTIATION

The task is adjusted for three different levels of ability, Starving Artist, Master Artist, and DaVinci. Copies of differentiated worksheets are available at `http://bit.ly/tattooworksheets`. Students at each of these levels are required to investigate the symmetry of both the mosaic and the Escher print. However, only the Master Artist and DaVinci levels are required to investigate how strategic coloring of the Roman Mosaic affects the original symmetry. Finally, the constraints for the third task of the student worksheet differ at each level, increasing in difficulty as the level increases. Starving Artists need only ensure that their designs have at least two types of symmetry, while Master Artists need to have at least four types, and finally, DaVinci students must have four each of rotational and reflection symmetry. Figure 7 illustrates possible solutions for an extension activity for the Master Artist and DaVinci levels. Adding more constraints to the activity increases the difficulty level of the task. For instance, teachers may wish to ask questions such as the following to differentiate the task for accelerated learners, "How could you color this to eliminate exactly 1 rotational symmetry?" Teachers could present this activity to Starving Artist students who are waiting on peers to finish.

7 THE RESULTS

The beauty of GeoGebra is that it encourages even the least artistic students to create impressive designs. I found myself gaining confidence in my abilities by using the different options, such as the polygon, freehand, and pen tools to create something simple. Then, I was able to reflect or rotate my creation about a line or point to result in a sophisticated, ornate design that I would never have imagined in the beginning. I also experimented by starting with a regular polygon, dividing it up into congruent triangles in which I drew my own simple design using the pen tool. I then reflected the design across the triangles' sides to create symmetry. Quite by accident, I discovered that if I dragged one side of the polygon, I could morph the design into something completely new. These techniques are suggested in Figure 8.

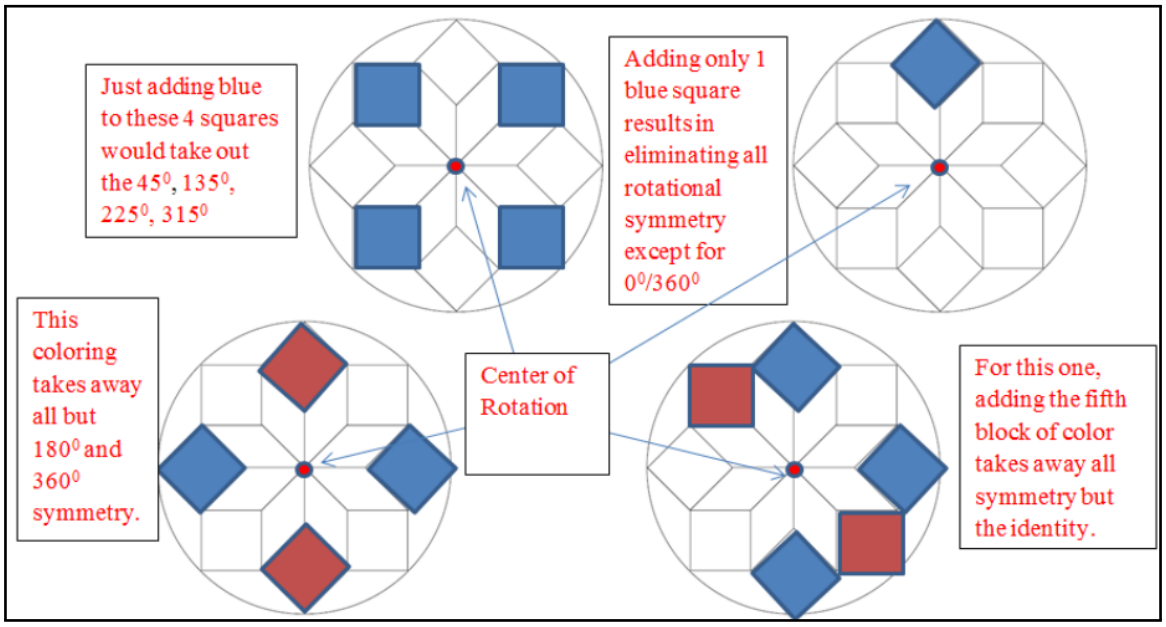

Figure 7. Possible solutions for an extension activity for the Master Artist and DaVinci levels.)

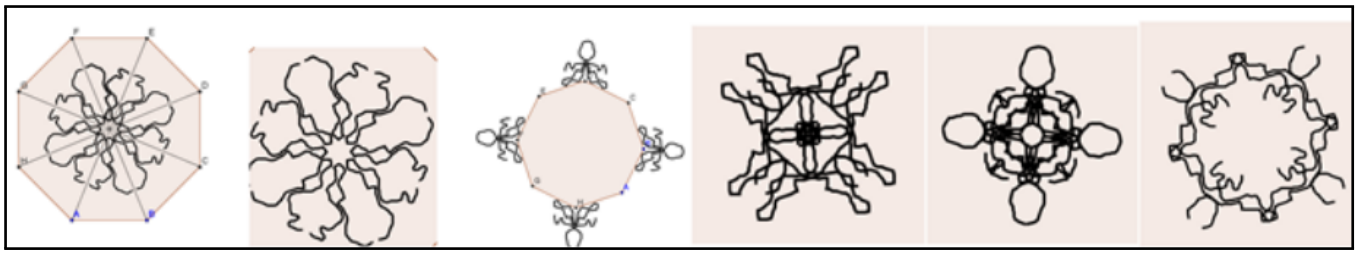

Figure 8. Designs constructed by scribbling inside GeoGebra.)

The design on the left of Figure 8 is the original design that was created by scribbling inside a triangle using the pen tool and then reflecting. The five right-most images in Figure 8 were created by simply dragging one side of the polygon with the inscribed original design.

8 Where do we go from here?

While planning this task, I discovered that the deeper I dug into the content and played with the sketches, the greater the possibilities and questions that arose. For instance, do the morphed designs have the same symmetry as the original? Are some reflections or series of reflections equivalent to a rotation and vice versa? I am hoping my students are as eager to ask more questions. However, this task has not yet been tested in the classroom.

A challenge to readers. I'd like to pose a challenge to you. Try the activities in your own classrooms and share your results. Tell me what worked and what didn't. Share any student questions that arose which surprised you. According to the NCTM, "Reflection and analysis are often individual activities, but they can be greatly enhanced by teaming with an experienced and respected colleague, a new teacher, or a community of teachers" (NCTM 2000, p. 19). Having said that, I eagerly await your feedback and suggestions. Happy sketching!

9 RESOURCES

The author has made the following resources available to interested readers.

- Student Worksheets (`http://bit.ly/tattooworksheets`)
- Teacher Notes with Answer Keys (`http://bit.ly/tattooteachernotes`)
- GeoGebra Sketches (`http://bit.ly/Foosgeogebrabook`)

REFERENCES

[1] GeoGebra. Available on-line at http://www.geogebra.org/cms/en/

[2] Martin, W. G. (2011). *Taking a Spin*. Retrieved July 13, 2014, from `http://www.nctm.org/uploadedFiles/Journals_and_Books/Books/FHSM/RSM-Task/RSM_Task-Taking_a_Spin.pdf`

[3] Math Assessment Resource Service. *Roman Mosaic*. Retrieved June 27, 2014, from `http://map.mathshell.org/materials/download.php?fileid=1163`

[4] National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

[5] National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Author.

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