SYSTEMS OF INEQUALITIES: USING GEOGEBRA TO PLAN YOUR PARTY

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Abstract

In this article, we describe an activity that provides students with opportunities to explore systems of inequalities in engaging and personally meaningful ways. Students use sliders and the graphing capabilities of GeoGebra to explore various pricing options for a school fundraiser.

Keywords: geometry, transformation, connections, Common Core

1 INTRODUCTION

During my second summer of graduate classwork, I took an advanced teaching methods course with 9 other practicing secondary mathematics teachers. The goal of this course was to learn about rich tasks and their usefulness for teaching students various mathematical concepts. Hewson [1] lists the following benefits of rich tasks. "Rich tasks can enable students to work mathematically by allowing them to:

- Step into activities even when the route to a solution is initially unclear
- Get started and explore because the tasks are accessible to pupils of wide ranging abilities
- Pose as well as solve problems, make conjectures
- Work at a range of levels
- Extend knowledge or apply knowledge in new contexts
- Work successfully when using different methods
- Broaden their problem-solving skills
- Deepen and broaden mathematical content knowledge
- See and make sense of underlying principles or make connections between different areas of mathematics
- Work within include intriguing contexts
- Observe other people being mathematical or see the role of mathematics within cultural settings" [1, p. 1].

This information, along with a selection of other readings and in-class discussions, assisted me in understanding the definition of a rich task. In the course, I learned how to write open-ended questions to provide my students with greater opportunity to think for themselves while strengthening their own mathematical understandings. I also learned how to write questions which require higher levels of thinking. Using these new skills, I created an open-ended student project, differentiated for three levels of learners. The materials utilize students' prior mathematical content knowledge as they explore concepts surrounding systems of inequalities collaboratively in small groups of three or four students.

2 THE TASK

Below is the task as laid out for my level two (i.e., "regular") students.

You are in charge of a fundraiser to earn money for a spring dance. The class decides to hold a chicken BBQ and Bingo. The goal set by student council is to raise at least $4000. They hope to raise at least $3500 on the chicken BBQ and $500 on Bingo. The school is supported by a community with the following characteristics.

- 500 adults
- 750 children
- Not everyone will be able to make it to the fundraiser because of ball games and other community events.

Your goals are to:

- Set the price for adult and children BBQ tickets, and also for adult and children Bingo cards.
- You then need to set a goal of how many tickets of each need to be sold keeping in mind that you will sell the same amount of tickets to children and adults for each event.
- You can always make more money than the goal, but not less. You don't want to disappoint your classmates and their dream for a spring dance.
- Give the actual amount of money you will make for each event based upon the number of tickets you sell. Show the math behind your answer, and elaborate on your reason behind the prices and number of tickets sold.

3 ALIGNMENT TO COMMON CORE

The task addresses standard A-CED, "3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods" [3, p. 65].

4 IMPLEMENTING THE TASK

Groups of students are given access to the GeoGebra [2] applet illustrated in Figure 1. (*Editors' note*: Readers can interact with an active version of the applet at `http://www.geogebratube.org/student/m129254`). Once students launch the applet, they manipulate the four color sliders to

determine the best price for the Bingo cards and BBQ tickets (adult and child pricing). The sketch also displays inequalities represented by the values set by the sliders.

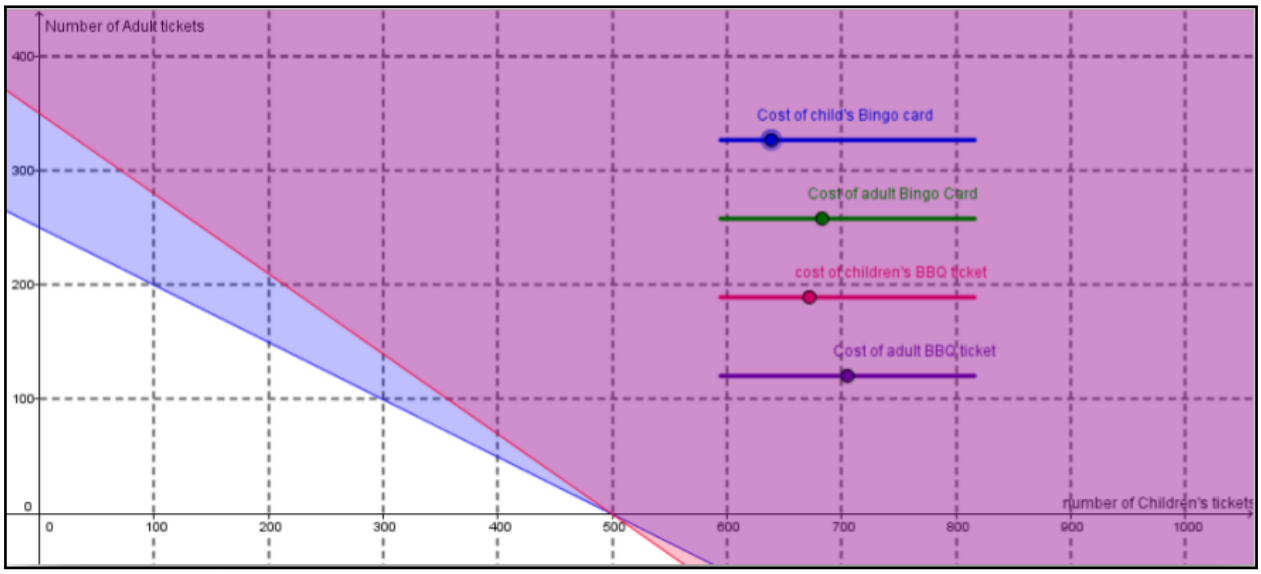

Figure 1. GeoGebra sketch depicting various ticket sales scenarios.

Figure 2 illustrates the values for the cards, tickets, and inequalities which are controlled by the sliders.

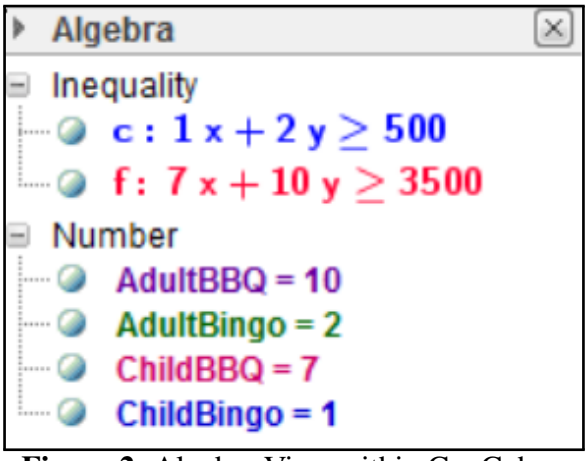

Figure 2. Algebra View within GeoGebra.

The GeoGebra tools assist students as they establish the best price for each event that satisfies the earning goal specified in the problem. Once a team agrees upon the event pricing, members set a target for the number and type of each ticket/card to be sold. Once again, the GeoGebra sketch provides useful information. For instance, numbers of children's tickets are represented along the x-axis; adult tickets, along the y-axis.

Once the group agrees upon the number of tickets sold and the pricing, they have to provide evidence that they have attained the objective as laid out in the activity. Students' final task is to represent the math behind their investigation using symbolic mathematics. Students may use variables to represent the number of tickets and cards sold or the cost of each item. Figure 3 provides an example of student

work on this portion of the assignment. Once they have completed this last step, each individual group should return a ticket/card pricing and sales objective which will be unique for their fundraising.

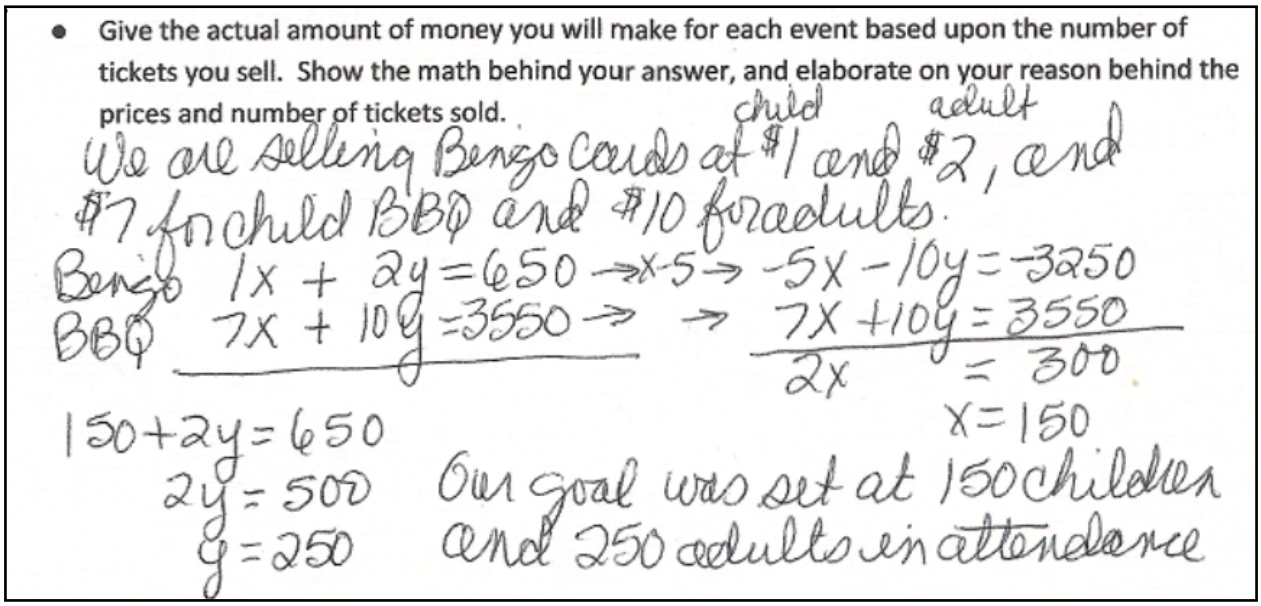

Figure 3. Algebra work supporting ticket and card pricing.

This fundraising task consists of two parts in two pages. The first part describes cost and attendance constraints for the party; the second part provides students with follow-up questions to be explored with the GeoGebra applet (*Editors' note*: A PDF version of the worksheet is available for download on the *North American GeoGebra Journal* website, `http://www.geogebrajournal.com/index.php/ggbj/issue/view/4`). Follow-up questions intended for "typical" students are given below.

Open the Systems of Equations GeoGebra file:
`http://www.geogebratube.org/student/bHWETXS0K#material/113115`
Take turns sliding the pink and blue sliders, and then answer these questions

1. How are the sliders related to each other? What do the $m1, m2, b1$, and $b2$ represent?
2. How would you adapt the sliders to create a different type of solution to the system of equations?
3. Create a system of equations in which the lines do not cross. What conclusions can you draw from your answer?
4. Can you create a system of equations that has more than one solution? If so, explain how you would go about doing this. If not, explain why you cannot.
5. Can you add a third line to your system of equations in which it will share the same solution as the other two lines in the system? If so, how would you go about finding the equation of that third line? If not, explain why you cannot.

5 DIFFERENTIATION

We've differentiated the Party On task across three levels of students (struggling, "typical," and accelerated). Struggling students are given a more restrictive customer base (500 adults and 350 children). The number of potential adult and children customers is displayed in the linked GeoGebra applet. For the accelerated students, the customer base is given in terms of family make-up. These students are also provided with a fractional relationship representing the age breakdown of the students in the area (300 families with 2 parents and 2 children, 150 families with one adult and 3 children, 1/3 of the children are aged 6 and under. One-third of the children are between the ages of 7 and 12, and the rest are over the age of 13).

6 CONCLUSION

Rich tasks are a great way to assist our students with thinking outside the box and enable them to control their learning. As a teacher, it is important for us to steer away from the closed minded questions and single answer worksheets, to ones which will encourage our students to connect to the math and not simply regurgitate it. GeoGebra is just one tool which can be used to put learning in the hands of our students.

REFERENCES

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