Computer as the sole drawing tool and school results in geometry

Jiri Vanicek and Josef Lombart

Abstract: A factor that plays a significant role in the effective use of information technologies is the teacher's belief regarding the appropriateness and usefulness of such teaching. The research presented here addresses the following question: How does replacing ruler and compass constructions with constructions using Geogebra impact the geometrical knowledge of school-aged students? Twelve year-old subjects from six different schools worked through identical, 8-day geometry units. After each unit was completed, students took two tests - one focusing on pencil-and-paper drawing (i.e., the drawing test), the other consisting of short answers (i.e., the non-drawing test). In the paragraphs below, we present the content of our teaching experiment while discussing the promise and the perils of an GeoGebra-centered approach to classic geometry constructions. We also share data gathered from a teacher attitudinal survey. Our research suggests that a teacher's perception of efficiency of GeoGebra use depends on beliefs regarding geometry as either practical drawing or, alternatively, as the study of shapes and space in the world.

Keywords: Geometric Constructions, Geometry, GeoGebra

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1. INTRODUCTION

School geometry in the Czech Republic is closely connected to ruler and compass constructions. By-hand constructions and drawing play an integral role in constructive geometry in our schools; such tasks are included in all textbooks for schools, and the deeply rooted tradition has not changed - even in the computer era. This is evident in new sets of multimedia textbooks adopted in Czech schools (Fraus, 2011). Given this emphasis on geometric construction and drawing, it is somewhat surprising to note that many lower secondary school teachers have yet to recognize the benefit of the computer as a worthwhile tool in the teaching and learning of school geometry. In informal conversations with teachers, many express fear that the computer will do too much work for the pupil. Their concerns are based largely on previous experiences with graphing calculators that, in their belief, undermined their students' abilities to carry out numerical calculations on their own, which they regard as a mistake. In an analogous fashion, many teachers believe that compass and straightedge constructions on a sheet of paper are the means leading to understanding geometry. Pencil and paper drawing is supported not only by teachers using traditional teaching methods but also by the teachers who have adopted constructivist-oriented teaching methods - those who prefer (re)construction of a piece of knowledge by the pupil to the transfer of knowledge from teacher to student. Both traditional and reform-oriented teachers regard the development of drawing and geometric construction skills as a primary goal of geometry instruction. Teachers whose teaching methods are based on the constructivist theory stress manipulation with objects and regard drawing as a principal means of building new knowledge (Prucha, 2003) and developing mental models of newly constructed concepts (Hejny, 1995). Teacher attitudes regarding geometric construction and drawing have the potential to hinder the use of computer as an environment for student discovery, hypothesis building and testing, and teacher scaffolding of various geometry-oriented tasks (Van der Stuyf, 2002). Teachers, particularly those with limited personal experience using the computer as a teaching and learning tool in mathematics education, prefer the use of pencil and paper drawing to GeoGebra. Is this a sign of inertia on the part of teachers alone? Or is teachers' reluctance to use technology with students supported by mathematics education research? Our thorough search in Czech pedagogical literature discovered few, if any, answers. The Czech official document Framework Education Programme for basic education does not mention compulsory pencil and paper drawing as the tool of geometrical teaching (FEP, 2007).

2. RESEARCH QUESTIONS

Student Performance. Given the situation in Czech schools, we wondered if it would be feasible to replace pencil-and-

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paper based geometry constructions with computer-based constructions using GeoGebra.

- Will students who use GeoGebra in lieu of more traditional paper-and-pencil construction methods perform at comparable levels?
- Will GeoGebra undermine students' ability to perform traditional compass and straightedge constructions?
- Are there particular problems that are better-suited for students in a traditional pencil-and-paper audience? A GeoGebra construction audience?
- Does the source of such differences lie in the computer application itself and its changed representation of geometry or are differences between the groups merely the side effect of the use of computer in teaching?

The computer shows geometry differently, particularly in dynamic environments where objects may be dragged, and this may impact student understanding in unforeseen ways. Furthermore, instructional methods employed by teachers using GeoGebra will necessarily be different from those in traditional classrooms. These methods may lead to student misconceptions. Given the relative inexperience of teachers and students using software - and the lack of teaching materials available to learn geometric constructions using GeoGebra - we anticipated that the use of computer as the sole drawing tool when teaching geometry would negatively impact student performance.

Student Attitudes. It is also important to study the pupils' and teachers' view of teaching and mathematics when pencil-and-paper drawing is completely replaced by the computer.

- In which problems do pupils appreciate the use of computers most?
- Which "computer construction" problems should thus be included in school curricula?

3. RESEARCH METHODS

In an effort to answer the aforementioned questions related to student performance, we employed a quasi experimental design with control and experimental groups and a GeoGebra treatment. The control group performed geometric constructions in a familiar manner, with compass and straightedge. The experimental group, on the other hand, performed constructions wholly in a computer-based (GeoGebra) environment. After both groups completed a construction unit consisting of 8 lesson modules, the pupils were administered two tests. The first test (i.e., the drawing test) required students to construct various paper and pencil sketches (e.g., identify symmetry lines for objects with bilateral symmetry). In the second test (i.e., the non-drawing test), students wrote short responses from writing prompts dealing with geometry constructions. Students did not have access to computers as they completed either test. The tests were constructed specifically to compare the extent to which geometrical cognition is connected to compass and straightedge drawing. Special attention was paid to the method of testing. Introduction of computers brings new problems and new approaches that differ markedly from the traditional approaches. Computer supported teaching and learning usher in new teaching and learning paradigms. According to Kuhn (1996), the world views advanced by each paradigm are diametrically opposed. As such, one cannot be assessed on the basis of the other (Kuhn, 1996). As a consequence, it is problematic to compare the study results of experimental and the control groups. Specifically, it is difficult to determine to what extent traditional and new "computer based" problems should be included in assessments. It is inappropriate to assign problems that require computer-based methods to the control group. Ultimately, the experimenters administered assessments including only traditional problems, items that do not explicitly require computer-aided strategies. At this point, we were curious to determine if GeoGebra would undermine students' ability to perform traditional compass and straightedge constructions. If there is no significant difference in performance between the control and experimental groups, the findings will suggest no harm in using GeoGebra when studying geometric constructions.

4. RESEARCH DESIGN

Fast and accurate constructions on computer save time which can then be devoted to new teaching approaches, e.g., manipulation with a dynamic figure, discovery of properties and mathematical relations in experiments, formulation and verification of pupils' hypotheses, project teaching applying the acquired knowledge. This observation guided much of our work planning the curriculum for the teaching experiment. During the 8-day experiment, subjects studied various ideas related to congruence and symmetry, subject matter that is part of the ordinary curriculum for 12 year olds in the Czech Republic. Subjects in the experimental group explored this content in new ways. The following is a brief description of dynamic sketches / tasks explored by experimental group members.

- Congruence Sketches: Pupils manipulated pairs of congruent objects (e.g., ellipses, polygons). When one object was altered, the other changed accordingly. These sketches were designed to help students refine their notions of congruence (what congruence means and what congruency "looks like.");
- Non-Congruence Sketches: Pupils manipulated pairs of objects, a "preimage" and a non-congruent "image". When one object was altered, the other changed accordingly. These sketches were designed to help

students refine their notions of congruence by encountering non-examples;

- Symmetric Sketches: Pupils manipulated geometric shapes that were created with reflection transformations over one or more lines of symmetry. Students observed the changes in the position and the shape of the image when symmetry lines where moved. These sketches were designed to help students refine their notions of symmetry;
- Congruence Construction Sketches: Pupils first constructed specific shapes using the one-click-tools of GeoGebra then measured components of the shape to verify properties of the shape. Using these properties, pupils reconstructed the shape without using one-click tools and without teacher's instruction. These replication tasks were designed to encourage discovery of various traditional constructions (e.g., constructing a regular n-gon);
- Symmetry Line Sketches: Pupils were assigned an object and asked to construct its line of symmetry experimentally;
- Transformational Sketches: Pupils were provided with a sketch of an initial shape (i.e., a "preimage") and a congruent image generated by a composition of reflections over the x- and y-axes. Through dragging and experimentation, students were asked to determine a relationship between the position of the preimage and image. These sketches were designed to help students reframe their notion of congruence to include transformational perspectives.

The research was conducted in 6 schools with a total of 125 pupils in experimental sections and 128 pupils in control sections. The teaching in the control classes was traditional, without computer support, while instruction in the experimental classes took place primarily in computer labs. For the most part, teachers of the experimental sections had not used computers to teach mathematics prior to this study.

5. TEST RESULTS

After the 8-day treatment, two written tests were administered to each subject. Average test scores of the control and experimental sections were compared on a school-by-school basis using simple t-tests. Comprehensive testing was not possible in case of this method as it did not meet the condition of homogeneity of variance. For each test, the statistical hypothesis was tested against the so-called null hypothesis: There is no statistically significant difference between student performance in the experimental and the control groups. The Mann-Whitney U Test was used for the overall comparison of data. For the drawing test, we reject the null hypothesis (p=0.0289) and conclude that there is a statistically significant difference between tested groups. For the non-drawing test, we accept the null hypothesis (p=0.3054). Variance analysis enabled comparison of results at the two different types of schools (basic schools and lower secondary grammar schools). The result of this testing showed statistically important p-value indicating a considerable difference in the first (drawing) test (Fig. 1) both with the lower secondary grammar school students and basic school pupils.

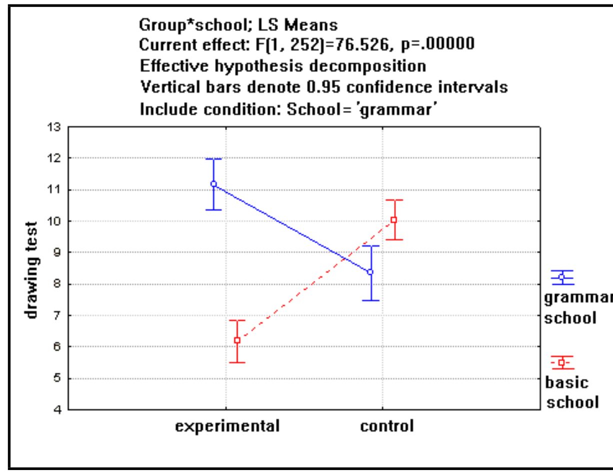

Fig 1: ANOVA results for student drawing test for grammar school and basic school students

6. ANALYSIS OF PUPIL SOLUTIONS

We analyzed students written work in an effort to determine how the use of GeoGebra affected student problem solving strategies. The difference between control and experimental groups was striking for problems in which computer environment presented the geometrical situation differently than the blackboard. For example, pupils from the experimental group were largely unsuccessful when asked to determine the angle bisector of given angle where one of the "lines" was deliberately markedly shorter than the other (Fig. 2). Because lines in GeoGebra always extend to the edge of the projection plane (i.e., the computer screen), students had difficulty interpreting "lines" that were represented in the manner depicted in Figure 2.

Students from the experimental group also found it difficult to construct a reflecting line from a given preimage and reflection image. We suspect that this is probably the result of overuse of automated reflection tools in GeoGebra. Denied the opportunity to construct reflections by hand, pupils seem to have failed to master important properties of line symmetry. No difference in reading ability (based on reading test instructions) was observed between the groups. However, one of the teachers claimed that pupils in the experimental group were so highly motivated to construct objects that they wanted to lose no time reading the assignment or descriptions of the objects. The motivation potential of the aid was counterproductive in this case.

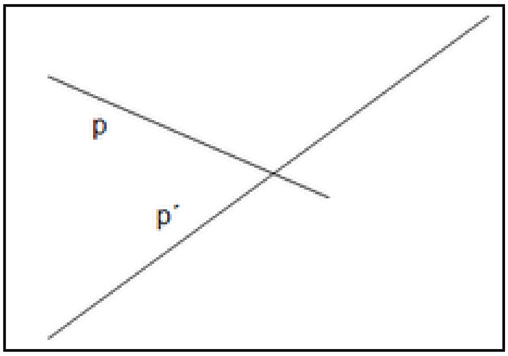

Fig 2: Angle bisector construction task with two intersecting lines, one of which is deliberately shortened

7. ANALYSIS OF PUPIL QUESTIONNAIRES

At the end of the study, students in the experimental sections were provided with an opportunity to reflect on their geometry construction experiences with GeoGebra in an attitudinal questionnaire. The questionnaire consisted of the open-ended, free-response questions listed below.

- Which of the activities with the computer have you enjoyed?
- Do you think learning with the help of computer was better/worse than normal?
- Do you think you learned more with the computer?
- What would you prefer to do with the computer in math next time?

The majority of pupils believe they have learned more with the help of ICT than they would have in traditional lessons (although this was not necessarily detected by our instruments). Most of all, pupils indicated that they wanted to improve their skills in work with and operating the software GeoGebra and in work with computers in general. Their responses suggested that they were less interested in the studied subject, but more interested in the teaching aid itself. Anecdotally, we observed a decline in students' interest in the software after our fifth lessons. As our lessons became progressively more difficult, involving more sophisticated knowledge of GeoGebra, student interest tended to wane. Many students expected that the computer would make their work easier. As our GeoGebra tasks became more challenging, the appeal of the software diminished.

8. ANALYSIS OF CLOSED INTERVIEWS WITH TEACHERS

In many ways, the teachers' comments about GeoGebra mirrored those of their students. Teachers appreciated that GeoGebra provided an alternative to traditional teaching that their students enjoyed. However, they also claimed that their pupils learned nothing more than they would have if taught traditionally. In closed interviews, many of the teachers expressed a believe that computer-supported constructions are not an adequate replacement for traditional teaching with pencil and paper. However, they did concede that GeoGebra can result in increased attractiveness of the subject matter, better visualization, and improved presentation of mathematical content. Teachers voiced concerns regarding the way in which GeoGebra represented formal geometric objects (e.g., lines with arrows). In their discussions, teachers focused their concerns squarely on traditional curricula without taking new didactical possibilities of computer learning into much account. On a related note, teachers noted a lack of support for computer supported teaching both in the school organization and in official state educational documents. Some teachers concluded, based on their experience and observations from this experiment, that mere turning over of traditional pencil and paper construction problems into computer environment bring no due educational effect. Clearly, to implement new computer-supported teaching approaches into classrooms in ways that impact student learning, significant teacher support and professional development are required.

9. LIMITATIONS

As we reflect on our experience incorporating GeoGebra into school classrooms, limitations of our work become more apparent to us. Of primary concern are the instruments we used to measure the impact of computer-aided instruction in school classrooms. For instance, although students reported that they had learned much while using GeoGebra, our tests did not adequately measure this. Several parameters of our study may have inadvertently led to a rejection of the null hypothesis (when rejection was not warranted):

- Problems solved in the research tests are constructed with ruler and compass, not computer
- Tests deliberately contain types of problems that are solved more easily with the help of the computer (problems with verification of pupils' hypotheses, problems with dynamics, generalizing problems)
- Teachers lack experience of computer supported teaching, are not efficient in managing teaching in a PC lab while teaching without computers is mastered by them routinely
- Pupils are beginners in work and operation in geometrical learning environments
- Teachers must first learn themselves to work with didactical software
- When using the computer, teachers must first begin to understand the changed teaching goals
- Teachers might be prejudiced against computer supported teaching

Other parameters may have inadvertently led to an acceptance of the null hypothesis (when acceptance was not warranted):

- Motivational factor of work with computers for pupils
- Effect of the "added value" (if a teacher is planning teaching with new methods, he/she devotes to it a lot of energy, pupils feel the teacher's increased interest in teaching and their work, which positively affects the quality of the educational process Ð one must ask here whether a teacher's boosted effort and energy in any direction that results in the teacherÕs increase of interest in his/her pupils does not lead to an improvement in the quality of the educational process)
- Possible extrinsic teacher's motivation (if the headmaster/headmistress appreciates when teachers innovate their teaching styles)

These factors influencing the research results caused that the disparity between different schools was greater than the difference between the experimental and control groups.

10. CONCLUSIONS

The outcomes of the testing suggest that if the objectives of school geometry are closely tied with drawing, then substitution of ruler and compass by computer results in significantly diminished student performance at the lower secondary school level. However, if geometry is perceived as understanding of shapes and space rather than drawing, then study results do not decline if traditional ruler and compass construction is replaced by computers. The analysis brings some recommendations for the teacher's work. If the teacher insists that his/her pupils master standard construction procedures, he/she naturally cannot show his/her pupils the use of a single button with the help of which this work is done automatically. Pupils must be guided, albeit in game-like activities, they must be assigned subtasks and controlled while carrying them out. Teachers must be trained not only in how a particular computer application works, what potential it brings, how to manage computer supported teaching and manage pupils in the lesson, but especially what teaching methods to choose, what the benefit of the selected method is and what the goals of a particular curricular topic of school geometry is. In this respect teachers also need support in official state educational documents.

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