WHERE'S THE FIREHOUSE?

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Abstract

In this article, we explore a rich task that integrates number lines, absolute value, and optimization. GeoGebra plays a critical role in students' conceptualization of the task, which involves placing a firehouse in the "best" location given certain parameters. The reader is invited to try these activities in his or her own classroom, and to allow students to explore possible extensions using dynamic geometry.

Keywords: GeoGebra, optimization, discrete mathematics, problem solving

1 INTRODUCTION

As area schools continue their transition to the Common Core and to Partnership for Assessment of Readiness for College and Careers (PARCC), mathematics teachers and their students are worried about the interpretation of test scores. The PARCC sample assessments mark a considerable change in emphasis from procedural fluency to reasoning and sense-making. These teachers and students are worried, and rightly so, as a curriculum built around procedural fluency and memorization will be of little use to students who are tasked with building viable arguments using data or generalized patterns.

What, then, are teachers to do? If students are going to be expected to "construct viable arguments and critique the reasoning of others" (Common Core State Standards for Mathematics, 2014), then teachers need to expose students to rich tasks that require reasoning and sense?making skills throughout the school year. Such tasks feature problems that have multiple entry points to a solution, are open-ended so that students choose their own solution strategy (as opposed to being told what pathway to take), and are engaging so that students share their thinking, justify their work, and grapple with alternate solution strategies (Hewson, 2014). This paper focuses on the implementation of one such rich task, as adapted from Course 2 of the COMAP Series *Mathematics: Modeling Our World*.

2 TASK

Students are given a number line, with houses represented as blue points. Houses can only be placed at integer values. *In "Linear Village", all of the houses are on one street. Your task is to place the firehouse in the best possible location.* The task is depicted in Figure 1 below.

The reader will quickly wonder what is meant by the "best" location. This is a good question for students to consider initially. There are at least two criteria for placing the firehouse in the best location. One is to minimize the **Total Firetruck Distance**, which is found by summing the distances

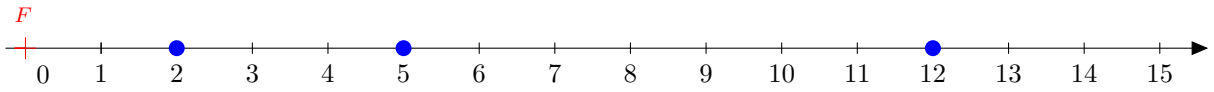

Figure 1. Firehouse Task. Place the firehouse, F, in the “”best possible location.

from the firehouse to all houses. Another is to minimize the **Maximum Firetruck Distance**, which is the distance from the firehouse to the furthest-away house. Both of these criteria are explored in more depth in a series of student worksheet and GeoGebra sketches I've created for students. I discuss these in the following section of this paper.

3 Teaching Materials

Depending on the level of the class and of the students, one of three worksheets:

1. `http://bit.ly/FHWS1`
2. `http://bit.ly/FHWSTwo`
3. `http://bit.ly/FHWSThree`

will accompany a *GeoGebraBook* (http://bit.ly/FHGeoGebra) created. Each worksheet is aligned with the *GeoGebraBook* and contains the applet shown in Figure 2, two additional applets, and an extension where students place their own houses to meet the given criteria for the location of the firehouse.

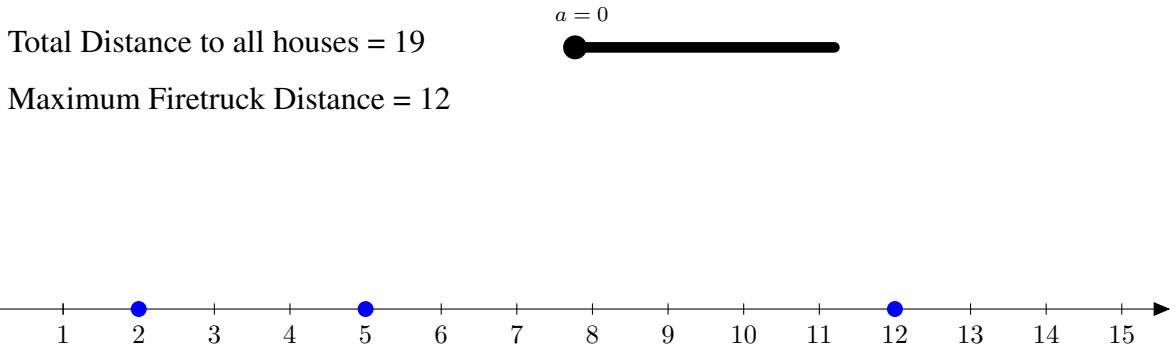

Figure 2. GeoGebra applets for the Firehouse task.

In the applet featured in Figure 2, students move the firehouse along the number line, trying to find the location(s) that minimize the Total Distance and the Maximum Firetruck Distance. In the initial applets, these calculations are made for the students. The worksheets mentioned above lead the students towards an understanding of the applet, then towards discovering the underlying mathematics and making conjectures about the best firehouse location(s).

In Figure 3 the final applet for a middle-level worksheet is illustrated. Instead of a number line, houses will be plotted on two "streets" meeting at a right angle. As before, the firehouse can still be placed at any integer value on either of the streets. The mathematics behind this picture is identical to the first applet, but students may not see this at first.

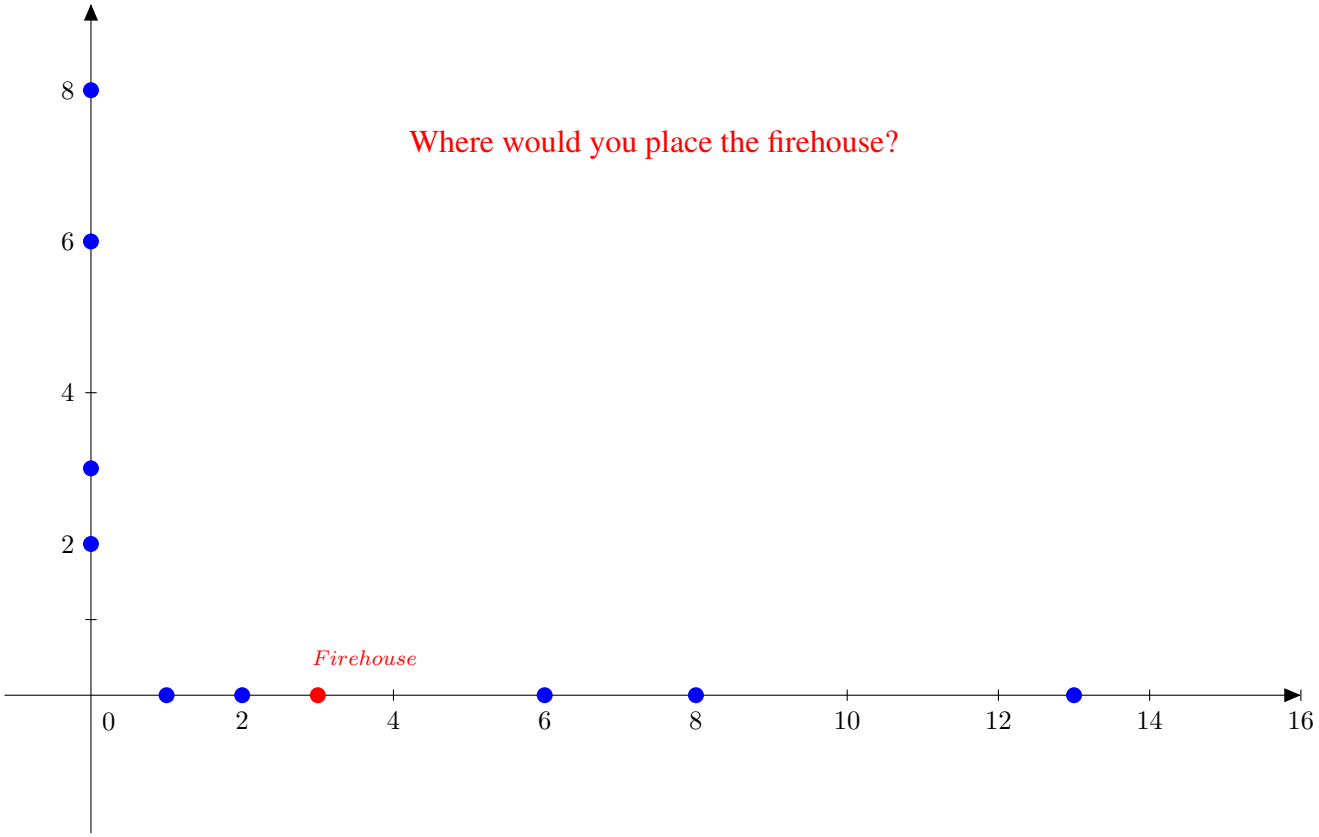

Figure 3. Middle-level extension the Firehouse task.

In the final extension, students are asked to find the best firehouse location, with houses placed in various places in the first quadrant. The firehouse now can be dragged to any lattice point in this quadrant, as shown below in Figure 4.

4 Solutions

As students work through the GeoGebra applets and the accompanying worksheet, they begin to build viable arguments using data. Many notice that as the firehouse is moved one unit, the firehouse gets closer to or further from each house. Once students see this pattern, they can begin to generalize patterns.

First, given a number line, the firehouse location that minimizes the Total Firetruck Distance occurs at the "median" house; the house whose value is the median of the set of values given. For example, in Figure 2 there are houses at location 2, 5, and 12. The location that minimizes the Total Firetruck Distance occurs at location 5.

If there are an even number of houses, then the firehouse can be placed at any location between the two median houses, inclusive. This is explained by creating an absolute value function to describe the

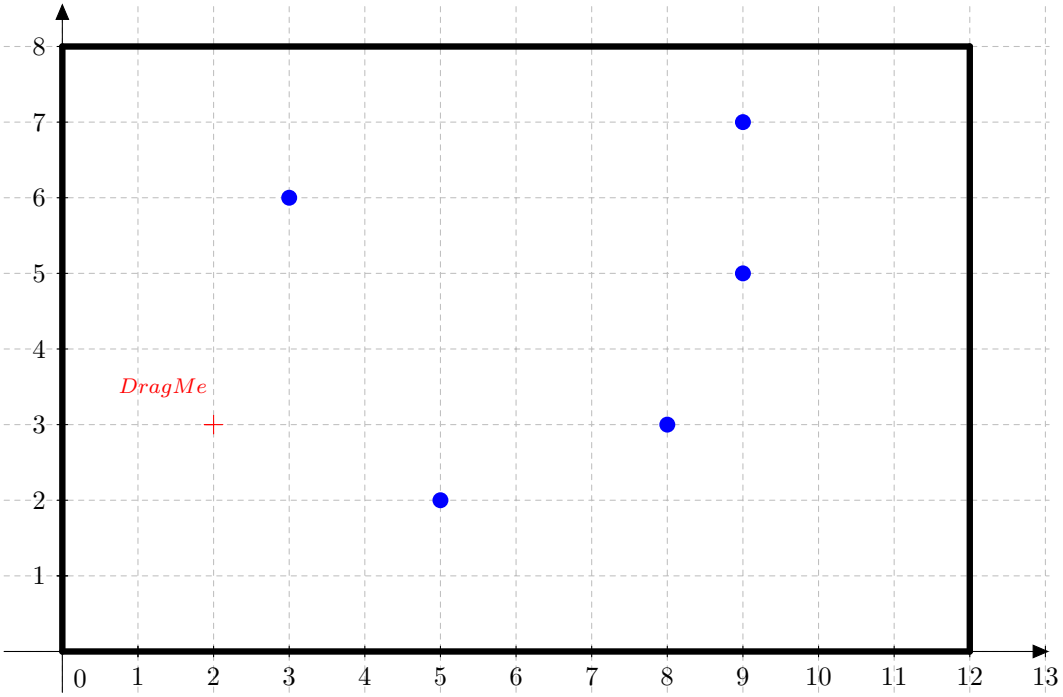

Figure 4. Upper-level extension of the Firehouse task.

relationship. The function describes the Total Firetruck Distance.

$$f(x) = |x - 2| + |x - 5| + |x - 12|$$

This function introduces students to piecewise functions in a natural way, as the function can be broken into four linear "pieces" as shown in Figure 5.

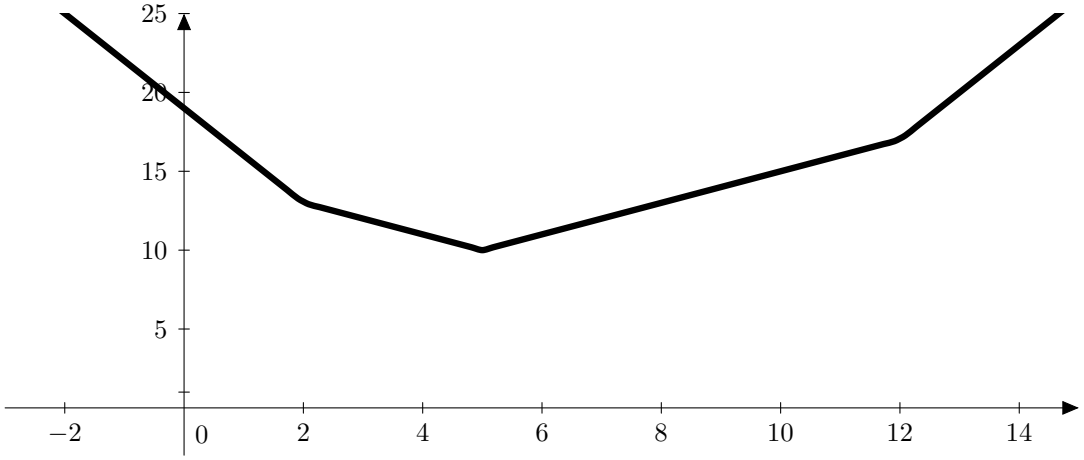

Figure 5. Sum of Absolute Value Expressions as Piecewise Function

Second, given a number line, the firehouse location that minimizes the Maximum Firetruck Distance (the distance to the furthest-away house) is the mean of the left-most and the right-most house. Students often will intuitively see this pattern.

5 CONCLUSIONS

Since there are multiple criteria for the optimal firehouse location, students have to justify which criterion is more important to them, leading to rich discussion of situations in which minimizing the maximum firetruck distance is more or less important than minimizing the total firetruck distance. An extension would be giving houses different weights; for instance, making one point a hospital that is 100 times more important than any one house.

Since students are not told what to do, they have to look for patterns and think about the context of the question as they solve the tasks. Students will also have to self?monitor; if they find themselves in a dead-end, they will have to try another method. These are all fruitful practices that will help students to grow in their reasoning skills. Not being told how to do the problem will cause some tension early on; however, the long?term gains far outweigh the short-term pains. Moreover, the reasoning skills sharpened by rich tasks such as this one will help prepare students for success on the PARCC assessments, and in their post-education lives.

REFERENCES

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