Using GeoGebra to develop primary school students' understanding of reflection

Xenia Xistouri and Demetra Pitta-Pantazi

Abstract: This paper presents a sequence of instructional activities with GeoGebra for the teaching of reflection in primary school. It aims to demonstrate the way in which GeoGebra can be used to design an instructional program based on the stages of the 5Es instructional model. The 5Es is a student-centered model for teaching which consists of five stages: engagement, exploration, explanation, elaboration, and evaluation. We discuss the way in which GeoGebra offers a rich environment to support the functions of all stages.

Keywords: reflection, primary education, GeoGebra, transformations, dynamic geometry

1. INTRODUCTION

Dynamic mathematics software (DMS) is a powerful tool for mathematical teaching and learning. Instruction with DMS offers unique opportunities for learning through exploration, creative problem solving, and self-guided instruction (Clements & Sarama, 2001).

In the context of a large scale project investigating the development of ability in geometric transformations, this article addresses a teaching experiment with the use of GeoGebra for the teaching of geometric transformations in the primary school. It is based on the activities of an instructional program that aims to promote the understanding of geometric reflection in primary school children. This program was designed using the 5Es instructional model (Bybee, 1997). The advantages of this model are that it provides a planned sequence of instruction that places students at the center of their learning experiences, encouraging them to explore, construct their own understanding of concepts, and relate this understanding to other concepts. Therefore, the aim of this article is to demonstrate the way in which GeoGebra can be used to design an instructional program based on the stages of the 5Es model.

2. LITERATURE REVIEW

Reflection is sometimes defined as a flip of an object over a line. This rather informal definition is the one that is usually used in primary education when introducing reflection. A more formal definition is that reflection is "one-to-one and onto mapping of all points in the plane to all points in the plane with a line as a set of fixed points; this set is called the axis" (Martin, 1982). This is the conception that we want students ultimately to achieve, since it may facilitate a better understanding of reflections in the coordinate plane, for example that reflection in the x-axis is $T(x,y) = (x,-y)$.

Some traditional methods for the teaching of reflection include the use of tracing paper, a mirror, and the use of the MIRA. These methods seem to drive students towards the more operational conception of flipping. In recent years, the limitations of traditional approaches to the teaching and learning of mathematics have been expressed (Rahim, 2002). Maragos (2004) argues that in a traditional geometry course, students do not experience the discovery of geometric relationships, nor invent any mathematics. Battista (2002) points out the importance of providing rich student-centered learning environments that give students opportunities to develop their geometrical thinking. Therefore, a significant issue in mathematics education is that the role of computers should not be limited to teaching the same things in a better way, but also in teaching better things. GeoGebra provides a resourceful teaching environment that offers educators the possibility to develop new ways to connect, extend, and enrich their instructional activities in order to promote students' understanding of mathematical concepts. But where do educators begin in order to do this? This paper proposes the use of the 5Es instructional model as a foundation for designing such an instruction (Bybee, 1997).

The 5Es model, based on the constructivist approach to learning (Bybee, Taylor, Gardner, Scotter, Powell, Westbrook, & Landes, 2006), is used extensively in science. It consists of five learning phases: engagement, exploration, explanation, elaboration, and evaluation. Each phase has a specific function, and contributes to teachers' coherent instruction and students' construction of better knowledge,

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attitudes, and skills. According to Bossé, Lee, Swinson, and Faulconer (2010), these process standards are very similar to those of NCTM. However, they explain the way in which the 5Es can be viewed sequentially, both as instructional stages and processes.

3. THE INSTRUCTIONAL PROGRAM WITH GEOGEBRA

3.1. Aims and Principles

The aims of the instructional program that we designed were for students to be able to: (1) recognize the role of both the variables of the orientation and distance of the line of symmetry in the change of position; (2) identify and describe the variables affecting a reflection; (3) construct the image of a reflection; (4) identify and describe the variables affecting a reflection in the coordinate plane; and (5) construct the image of a reflection in the coordinate plane. Note that the research design includes an introductory lesson on recognizing reflection and its properties, which is part of the 6th Grade curriculum in Cyprus. The curriculum indicators suggest that students at this age should be able to: (i) recognize shapes with symmetry and find the line of symmetry; (ii) draw and describe the result of geometric transformations; and (iii) predict and justify the results of transformations of two-dimensional shapes. Similar indicators appear in other countries' curricula. For example, the NCTM *Standard for Geometry* suggests that in grades 6-8 students should be able to: (i) describe sizes, positions, and orientations of shapes under transformation; and (ii) examine the congruence, similarity, and line or rotational symmetry of objects using transformations. The instructional principles followed for the design of this program are rooted in the principles that students learn better when they are: (1) actively engaged in the process of learning, hence this instructional program is intended for students who are working on their own screens; (2) investigating hypotheses and making conjectures, hence students are dealing with open questions and guided discovery; and (3) working in groups and discussing observations and conclusions with peers.

3.2. The Learning Phases

Phase 1 - Engagement

The teacher accesses students' prior knowledge through the task and engages them in a new concept through activities that promote curiosity and elicit prior knowledge. According to Bossé et al. (2010), key elements that may characterize the activities of this phase are: being in a state of disequilibrium; predicting; connecting past and present learning experiences; organizing student thinking; and generating curiosity.

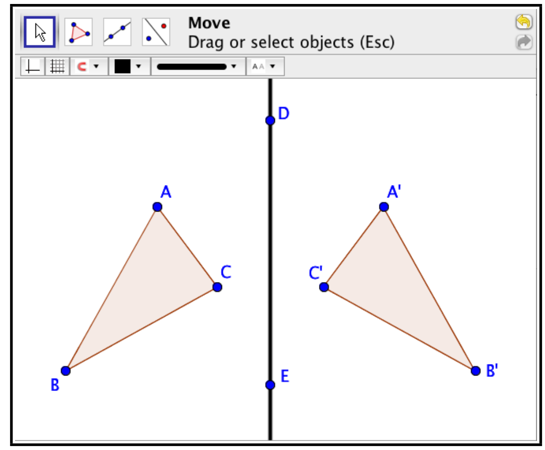

Fig 1: Phase 1 - Engagement activity

Figure 1 presents an engagement activity. The aim of this activity is to recall previous knowledge about recognition of a reflection based on its properties, and to generate curiosity about the role of the line of reflection in the change of position, regarding its orientation and distance from the pre-image. The first step of this activity is to ask students to construct the polygon and the line, and then to use the reflection tool from the menu to reflect the polygon over the line. Students are then asked to drag and rotate the line around the polygon, and to decide whether the relation between the two shapes remains invariant. Thus, they integrate their past learning experiences of recognizing a reflection and its properties, which is a key element in this phase. They are then asked to predict and ascertain when the smallest and largest distance between the image and pre-image occur. This directs their thinking towards the relationship between the two images, and not simply the role of the line of reflection. In the final step, they are asked to make predictions and find the circumstances under which the image completely overlaps the pre-image. For some students, this may cause disequilibrium, since in the natural world one's image in the mirror cannot overlap with oneself, and it may stimulate curiosity about the reasons for this phenomenon.

Phase 2 - Exploration

Exploration experiences provide a common base of activities within which current concepts, processes, and skills are identified and conceptual change is facilitated. The students may complete investigative activities that help them use prior knowledge to generate new ideas, explore questions and possibilities, and conduct a preliminary investigation. The key elements are: movement towards equilibrium; test and refinement of predictions and hypotheses; communication; mediation of investigation; and discovery of patterns and relationships (Bossé et al., 2010).

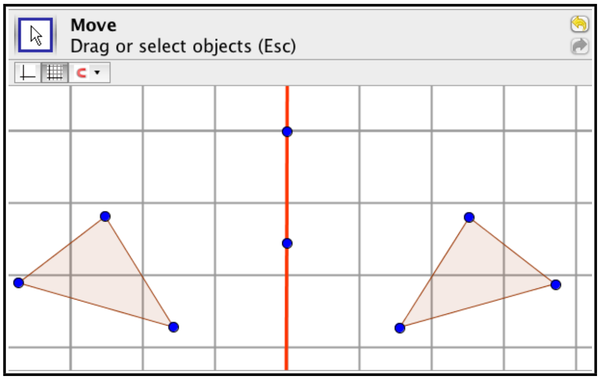

Fig 2: Phase 2 - Exploration activity

The activity designed for this phase (see Figure 2) requires the use of a pre-constructed GeoGebra file to explore a sequence of true/false statements regarding the direction of the image's position with respect to the pre-image (i.e. "The image can be above the pre-image"). The aim of the activity is to identify and describe the role of the line of reflection as a variable affecting the direction of the image's position with reference to the pre-image. The students are expected to drag around the reflection line and/or the pre-image to explore the statements. In this mediated investigation activity, students test and refine some given hypotheses. They may communicate with their peers in order to find possible strategies for investigation, such as changing the position of the line or of the pre-image. They are then asked to describe their strategies. This question leads them to discover the relationships between the reflection line and the images, as well as the patterns of how the position of the image changes according to the position of the line. This moves them towards equilibrium about the role of the reflection line regarding the direction of the image.

Phase 3 - Explanation

The explanation phase focuses students' attention on a particular aspect of their engagement and exploration experiences, and provides opportunities to demonstrate and express their conceptual understanding, process skills, or behaviors. It also provides opportunities for teachers to introduce a concept, process, or skill directly. An explanation from the teacher or the curriculum may guide them toward a deeper understanding, which is a critical part of this phase. The key elements are: formal and informal thinking; synthesizing ideas; model making; clarifying concept; formalizing language; and demonstrating conceptual understanding (Bossé et al., 2010).

For this phase, students are asked to use a pre-constructed GeoGebra file that presents a triangle (with fixed dimensions and position), a line of reflection, and the triangle's image. The aim is to identify and describe the role of the line of reflection as a variable affecting the distance between the image and the pre-image. In the first step, the students use the **Segment between Two points** tool to connect each vertex to its image. Then, they are asked to drag around the line of reflection and write down their observations about the way in which it relates to the segments. It is expected that some students will observe perpendicularity. Later they use the **Angle** tool to measure the angle and test their observations to confirm perpendicularity. In the next step, they use the **Distance or Length** tool to measure the distance from each vertex to the line of reflection, then drag the line of reflection around, and write down their observations about the distances. They are expected to observe that the distance between every point of the image and the line of reflection is always equal to the corresponding distance of the pre-image point. In this activity, students begin to shift from informal observations and language to formal observations and vocabulary such as 'distance,' 'measure,' 'angle,' and 'perpendicular' in order to describe and explain their observations. They begin to synthesize ideas and concepts from other contexts, such as length, angle size, and line relationships, to describe reflection. Hence, they begin to use formal language and different modes of expressing relationships and to develop a more conceptual understanding of reflection as mapping.

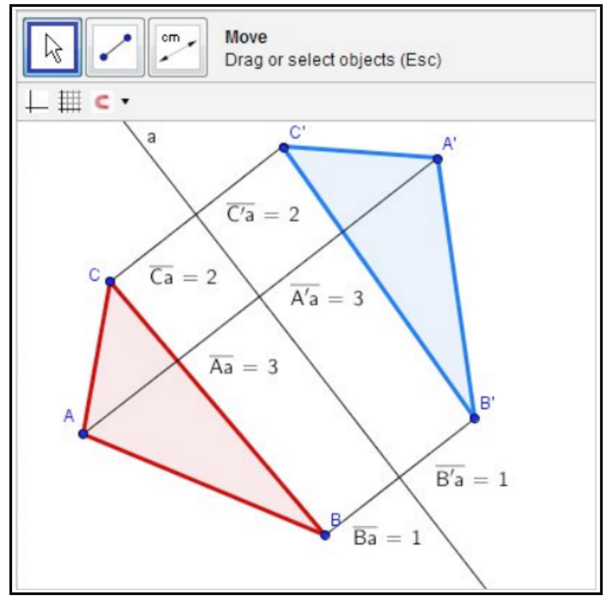

Fig 3: Phase 3 - Explanation activity

Phase 4 - Elaboration

Teachers challenge and extend students' conceptual understanding and skills. Through new experiences the students develop deeper and broader understanding. They apply their understanding of the concept through conducting additional extensive activities. The key elements here are: extending and/or applying understanding; checking peers' understanding; presenting and defending explanations; drawing reasonable conclusions; and formalizing language (Bossé et al., 2010).

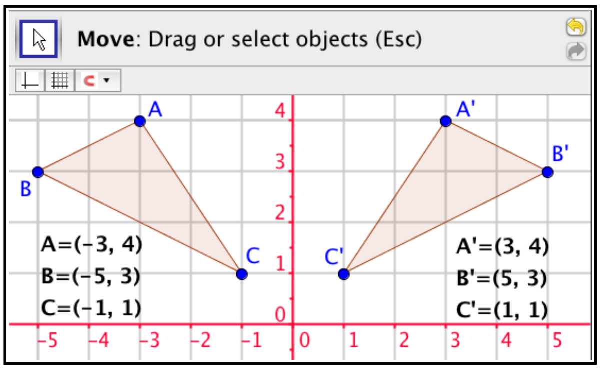

Fig 4: Phase 4 - First part of elaboration activity

In this activity (see Figure 4), students extend and apply their knowledge in the coordinate plane. In the first part, they are asked to use a pre-constructed GeoGebra file which presents a triangle and its image in the *y*-axis, and the coordinates of their vertices. The aim of the activity is to identify and describe the variables affecting a reflection in a vertical line in the coordinate plane. Students are asked to drag the vertices of the pre-image and write down their observations to describe the relationship between the coordinates of a vertex and those of its reflection image in a vertical line. This requires the use of some formal language and notation regarding the coordinates.

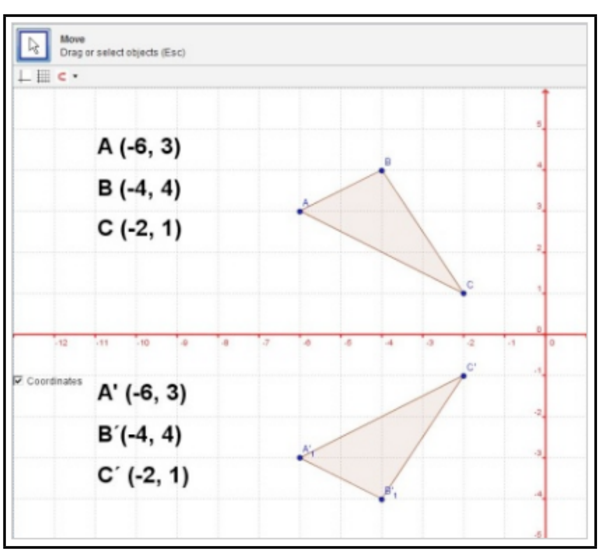

Fig 5: Phase 4 - Second part of elaboration activity

In the second part (see Figure 5), a similar pre-constructed file is given with a triangle and its image in the *x*-axis. In this file, a check-box option showing/hiding the coordinates of the image's vertices is also provided. The aim of this activity is for students to make predictions, justify, and verify hypotheses regarding the variables affecting a reflection in the coordinate plane, and to construct the image of a reflection in the coordinate plane. The students are first asked to predict the relationship between the coordinates of a vertex and those of its image in a horizontal line, based on their observations in the first part. They are then prompted to select the check-box to show the coordinates, and drag the vertices again to test their prediction. After this, they describe and explain their observations, using the formal notation of x and y. As a result, they draw reasonable conclusions and defend their explanations with the use of more formal language. They can also check peer understanding by hiding the coordinates again using the checkbox, then change the position of the pre-image, and ask their peers to predict and indicate the new coordinates that would construct the image.

Phase 5 - Evaluation

The evaluation phase encourages students to assess their understanding and abilities while providing opportunities for teachers to evaluate their progress towards achieving the educational objectives. According to Bossé et al. (2010), the key elements of the activities in this phase are: feedback; incentive for further investigations; reflection/self-examination; ability to answer open-ended questions; and evidence of development and change in thinking/behavior.

In this activity (Figure 6), students can self-evaluate and reflect on their understanding. The aim is to construct the image of a reflection. They are asked to imagine that GeoGebra does not have a reflection tool, and to describe the way in which they would construct the image of a segment/shape by using other tools (e.g., **New Point**, **Segment between Two Points**). Later, they use the **Reflect Object about Line** tool to reflect the image and verify their answer. If they make a mistake, they are prompted to reflect upon their possible errors and think about what they should have done differently. Therefore, errors can sometimes be used for future investigations (i.e. "Why did this happen? Try another case"). What is important is that students receive direct feedback.

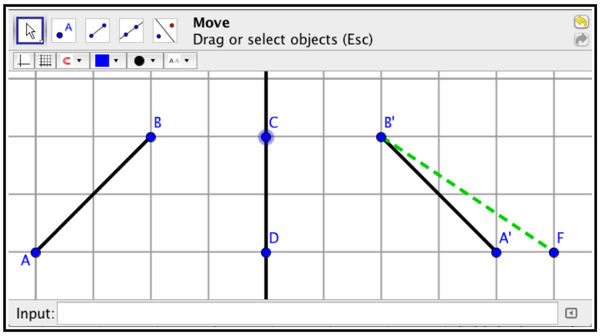

Fig 6: Phase 5 - Evaluation activity

4. CONCLUSION

It appears that GeoGebra is a valuable tool for students' active learning of geometric reflection concepts and properties at primary school. It offers a rich environment for designing a variety of activities with different cognitive requirements and objectives. In addition to this, it can support all the functions of the planned sequence of instructional stages based on the 5Es model of engagement, exploration, explanation, elaboration, and evaluation. This makes GeoGebra a valuable tool for designing a structured and effective instructional program for reflection in the primary school, and perhaps for teaching other geometrical concepts. This proposal requires further investigation.

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