A Laboratory Guide for Elementary Geometry using GeoGebra:Exploring the Common Core-Geometry Concepts and Skills

Jack D. Gittinger

Abstract: *A Laboratory Guide for Elementary Geometry Using GeoGebra* is a free, online collection of interactive, problem-centered, inquiry-based geometry lesson plans for upper elementary students that are based on the Essential Concepts and Skills for Geometry identified in the Common Core State Standards for Mathematics (grades 3-5).

Keywords: Common Core Curriculum, lesson plans, pedagogy, GeoGebra

1. INTRODUCTION

Currently, 44 states have elected to adopt the Common Core State Standards for Mathematics scope and sequence developed through the efforts of the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). This curriculum includes a significant and rigorous strand in geometry and measurement at the upper elementary grade levels (3-6).

Emerging computer applications such as GeoGebra offer the opportunity to engage students at all grade levels in highly interactive, inquiry-based problem solving within the study of geometry and measurement while developing technological literacy.

> Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software ... When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data ... They are able to use technological tools to explore and deepen their understanding of concepts. (NGA Center and CCSSO, 2010)

A Laboratory Guide for Elementary Geometry Using GeoGebra consists of a collection of 120+ interactive, problem centered, inquiry-based geometry learning activities for upper elementary students based directly on the essential concepts and skills for geometry and measurement identified in the Common Core State Standards for Mathematics for grades 3-6. It is not a stand-alone curriculum, but is intended to be used in conjunction with existing mathematics programs and textbooks. It is hoped that this laboratory guide will generate interest among elementary-level teachers in learning and using GeoGebra and its pedagogical philosophy of active engagement and inquiry in mathematics education.

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The materials discussed here are available on-line https://sites.google.com/site/geogebraiowa/elementary-geometry-project

2. THEORETICAL FRAMEWORK

In addition to content standards, the concepts and essential skills of geometry and geometric measurement that students are expected to master, the Common Core also provides a picture of effective mathematics education that is not limited to solely to content knowledge.

> The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important 'processes and proficiencies' with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and oneÕs own efficacy) (NGA Center and CCSSO, 2010).

A significant challenge, as any experienced teacher can tell you, is to meaningfully connect content standards with process standards. As teachers begin their efforts to align both their curriculum and classroom practices with the Common Core, they face challenges in rethinking how, in the case of this project, geometry and measurement can be taught in a manner consistent with the vision of the developers of the Common Core.

3. EXAMPLES OF ACTIVITIES FROM *A LABORATORY GUIDE FOR ELEMENTARY GEOMETRY USING GEOGEBRA*

Examine several of the key pedagogical practices suggested by the Common Core and how they can be reflected in lesson design. Each lab activity in the guide includes a student handout, essentially the lesson plan, and a related GeoGebra file.

3.1. Standards for Mathematical Practice: Teach so that students understand

> The Standards for Mathematical Content are a balanced combination of procedure and understanding. Students who lack understanding of a topic may rely on procedures too heavily . . . In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices (NGA Center and CCSSO, 2010).

Understanding does not arise from memorization. Understanding is developed through experiences with multiple models for representing concepts and encountering concepts in a wide variety of problem situations that serve to provide context for students. The GeoGebra lessons in the laboratory guide focus on promoting the understanding of concepts and procedures.

Figures 1 and 2 provide an example of a student activity sheet and accompanying GeoGebra sketch from the laboratory guide that supports the 'Teach so that students understand" standard. The activity, Volume and the Unit Cube, is designed to address Common Core Grade 5 Geometric Measurement.

> *Common Core Grade 5 Geometric Measurement*: Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
>
> - A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
> - A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of $n$cubic units.
>
> Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

3.2. Standards for Mathematical Practice: Teach so that students learn to make sense of problems and persevere in solving them

> Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem . . . They can understand the approaches of others to solving complex problems and identify correspondences between different approaches (NGA Center and CCSSO, 2010).

Iconic representations and manipulatives, whether physical or digital, are the "primary language" of elementary students exploring mathematical concepts and problem solving. Dynamic geometry software like GeoGebra provides teachers with a tool for creating problem situations that can be modeled, manipulated and reasoned through by students. A majority of the lessons in the laboratory guide represent learning through problem- or inquiry-centered instruction, with a problem or question driving the learning.

Figure 3 provides an example of a student activity from the laboratory guide, Area of Complex Shapes, that is designed to address Common Core Grade 3 Geometric Measurement. An accompanying GeoGebra sketch is shown in Figure 4.

> *Common Core Grade 3 Geometric Measurement*:
>
> - Recognize area as additive.
> - Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Level: 5
Folder: volume
File: 1_unit_cube.ggb

VOLUME AND THE UNIT CUBE

Background

One way to compare the size of containers is by ***volume***.

Volume is measured using unit cubes. On the GeoGebra page are eight unit cubes. Each edge of the cubes is one unit long. The unit could be inches, meters, yards, or miles – whatever unit is appropriate.

A solid or 3-dimensional figure which can be packed without gaps or overlaps using X unit cubes is said to have a volume of X cubic units. For example, if a solid figure or container can hold 5 unit cubes, its volume is said to be 5 cubic units.

Step 1. Practice

Use the MOVE tool to practice moving a unit cube. Each unit cube has a red point. Move the unit cubes by grabbing this point and dragging them to a new location.

Step 2. Measure

Find out how many unit cubes it takes to fill each of the 5 containers. Fill in the Data Table with your results.

Data Table	
Container	**Volume in Unit Squares**
red	
blue	
green	
black	
purple	

Figure 1: *Volume and the Unit Cube* Student Handout

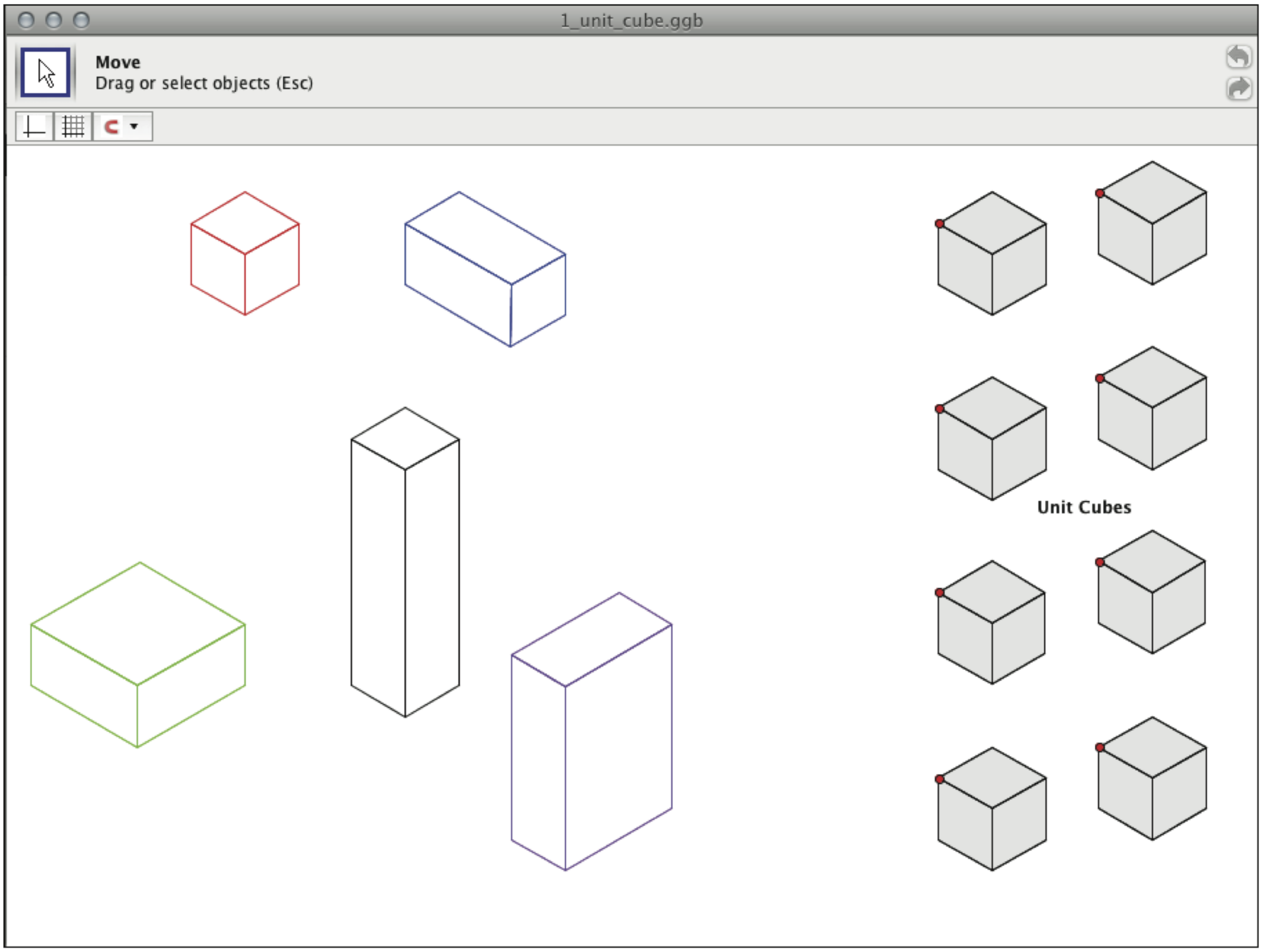


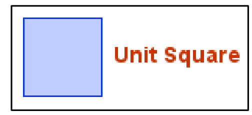
Figure 2: *Volume and the Unit Cube* GeoGebra sketch

Level: 3
Folder: Area and Perimeter
File: 7_area as additive_2

MORE PRACTICE IN DISCOVERING STRATEGIES FOR FINDING THE AREA OF MORE COMPLEX SHAPES

Introduction and Background

As you now know, the area of a rectangle is determined by finding out how many unit squares it takes to cover the rectangle.



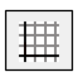
You have also discovered that the area of a rectangle can be computed by using multiplication. This activity will apply that discovery to a problem.

Step 1. The Problem

We need to measure the area in unit squares of the odd shape shown on the *GeoGebra* page.

Dragging the Unit Square around on the shape to measure the area has some problems. It is a large, complex shape and it would be easy to make an error.

Step 2. Estimate Area



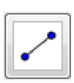
With the GRID turned off and without using the Unit Square, estimate the area of the large shape on the GeoGebra page.

Record your estimate: ________________________ unit squares

Step 3. Determine the Area

Can you think of a strategy for finding the correct area of this large, complex shape that might make your task easier and less likely to lead to error?

Use what you know about the area of rectangles to help you.



Use the SEGMENT BETWEEN TWO POINTS tool to add any lines to the shape that might help you divide it into smaller, easier to work with shapes. *Hint*: think of the shape as a connected set of rectangles.

Use the Unit Square or the GRID or multiplication to find and record the area of each of the smaller rectangles you created. Then add them together to find the area of the complex shape:

Step 4. Print and Share

Print your modified shape. On the printout, briefly describe your strategy. Compare your strategy for finding the area with that of a classmate.

Figure 3: *Discovering Strategies for Finding Areas of More Complex Shapes* Student Handout

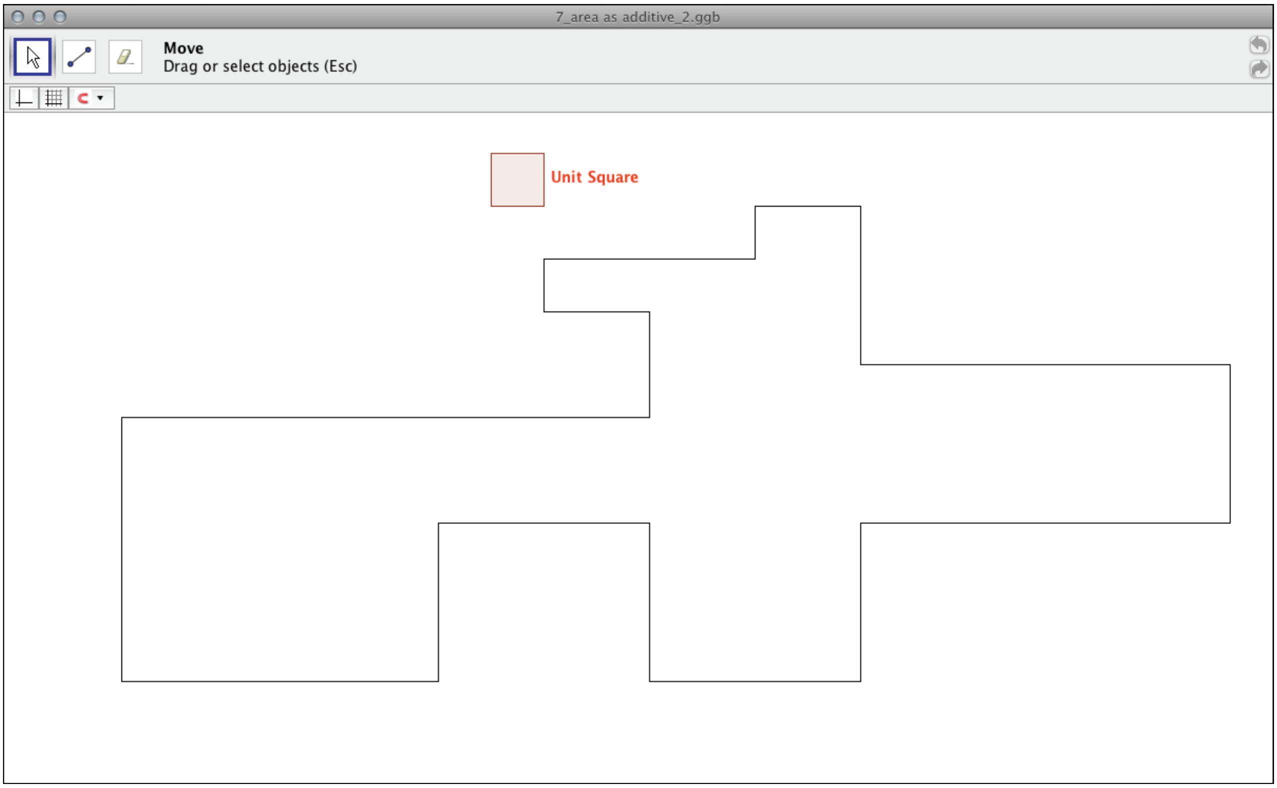


Figure 4: *Discovering Strategies for Finding Areas of More Complex Shapes* GeoGebra sketch

3.3. Standards for Mathematical Practice: Teach so that students learn to reason abstractly and quantitatively

> Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects (NGA Center and CCSSO, 2010).

Dynamic geometry software like GeoGebra provides students with an interactive environment in which they can quickly and easily create, manipulate, measure, and analyze digital representations of key concepts from geometry and measurement. This environment encourages students to move from simply trying to apply memorized or scripted methods of problem solving to reasoning based on conceptual understanding. The activities in the laboratory guide encourage students to solve geometric problems not by applying memorizing formulas but by using analysis and reasoning supported by the GeoGebra tool box.

Figure 5 provides an example of a student activity from the laboratory guide, Surface Area of Pyramids, that is designed to address Common Core Grade 6 Geometry. An accompanying GeoGebra sketch is shown in Figure 6.

> *Common Core Grade 6 Geometry*: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures.

3.4. Standards for Mathematical Practice: Teach so that students can construct viable arguments and critique the reasoning of others

> Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades (NGA Center and CCSSO, 2010).

Many if not most of the activities in this guide can be used to encourage students to search for strategies and methods of problem solving or to inductively develop understanding of a concept or procedure, and then compare and contrast their finding with their peers, either in small groups or with the whole class. For elementary mathematics students, this sharing of ideas and strategies represents the first step in what will later become a more formal process.

Figure 7 provides an example of a student activity from the laboratory guide, Maximize the Area, that is designed to address Common Core Grade 3 Geometric Measurement. An accompanying GeoGebra sketch is shown in Figure 8.

> *Common Core Grade 3 Geometric Measurement*: Recognize area as an attribute of plane figures and understand concepts of area measurement.

3.5. Standards for Mathematical Practice: Teach so that students can model with mathematics

> Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace . . . (NGA Center and CCSSO, 2010).

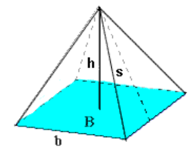
As previously noted, most of the activities in the laboratory guide are either problem-driven or involve using geometric constructions to solve authentic problems. As the guide continues to evolve, the addition of more lessons connecting geometry to "everyday life, society, and the workplace" is a priority.

Figure 9 provides an example of a student activity from the laboratory guide, Practice Using Paired Numbers, that is designed to address Common Core Grade 5 Geometry. An accompanying GeoGebra sketch is shown in Figure 10.

> *Common Core Grade 5 Geometry*: Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Level: 6
Folder: Nets and Area
File: 2_surface area_pyramid_using_net.ggb

FINDING THE SURFACE AREA OF 3-D SHAPES: THE PYRAMID



Introduction

Finding the total area of the surface of a 3-D shape like a pyramid can be very useful. It can answer questions like, "How much paint do I need to cover the surface?" or "How much wrapping paper do I need to wrap this container?"

There are formulas that you can memorize and use when needed. However, it is easy to forget a formula that you only use once in a great while.

Is there another way to solve a surface area problem without memorizing the formula? In this *GeoGebra* lab you will explore another method for finding surface area.

Step 1. Create and Experiment a Net

Use the MOVE tool and the sliders for l (length of base side) and ***h*** (height of pyramid) to change the dimensions of the pyramid. ***S*** represents what is called the slant length of the side of a pyramid.

Now slowly use the **Unfold / Fold the Net** slider to create a ***net*** of the pyramid. A net is created by imagining that you cut and unfold the 3-D shape into one, flat shape. Using the slider, the net can be folded back into the original 3-D shape.

Step 2. Find the Surface Area

Move the **Unfold / Fold the Net** slider all the way to the right to completely unfold the pyramid and display the net.

The net for a pyramid is a connected set of what two types of polygons?

You already know the method for calculating the area of a square and a triangle. Use those methods to find the surface area of the entire pyramid. Explain your solution:

Once you have computed the surface area of this pyramid, click on the **Check Surface Area Solution** checkbox to see the surface area as computed by *GeoGebra*.

Step 3. Practice

Turn off the **Check Surface Area Solution** checkbox.

Use the sliders to create three pyramids with sides of differing dimensions. Use the net to help you calculate the surface area of each new pyramid. Use the **Check Surface Area Solution** checkbox to check your solutions.

Height (h)	Length of Side of Base (l)	Slant Length (s)	Computed Surface Area	*GeoGebra* Surface Area

Conclusion

For a pyramid, the "official" formula is**: surface area = (l x 2) + (4) x (1/2 x l x s)**
But if you forget the formula, you can always use reasoning and a net to help you compute the surface area.

Figure 5: *Finding Surface Areas of 3D Shapes: The Pyramid* Student Handout

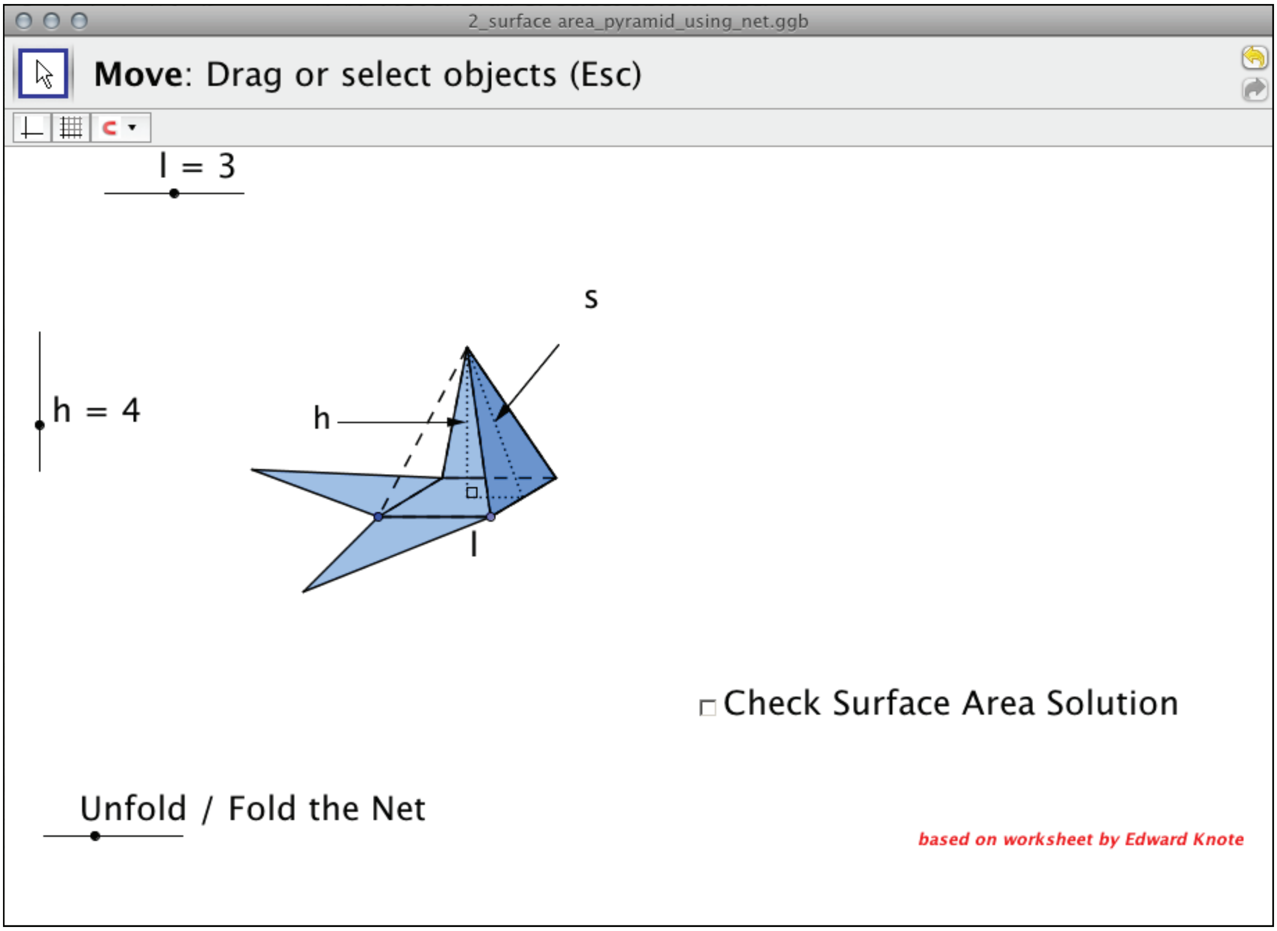


Figure 6: *Finding Surface Areas of 3D Shapes: The Pyramid* GeoGebra sketch

Level: 3
Folder: Area and Perimeter
File: 27_maximize_area.ggb

MINIMIZE - MAXIMIZE THE AREA

(Adaptation of Worksheet by Anthony Or. Education Bureau, Hong Kong)

Challenge 1

Use the MOVE tool to drag the matches and form a polygon. You can rotate the matches by moving the red end.

Minimize
What is the smallest area in square units you could construct?

__________________________ Square units

Challenge 2

Maximize

What is the largest area in square units you could construct?

__________________________ Square units

Follow-up

Check with your classmates. Did anyone find a smaller or a larger polygon?

Figure 7: *Maximize the Area* Student Handout

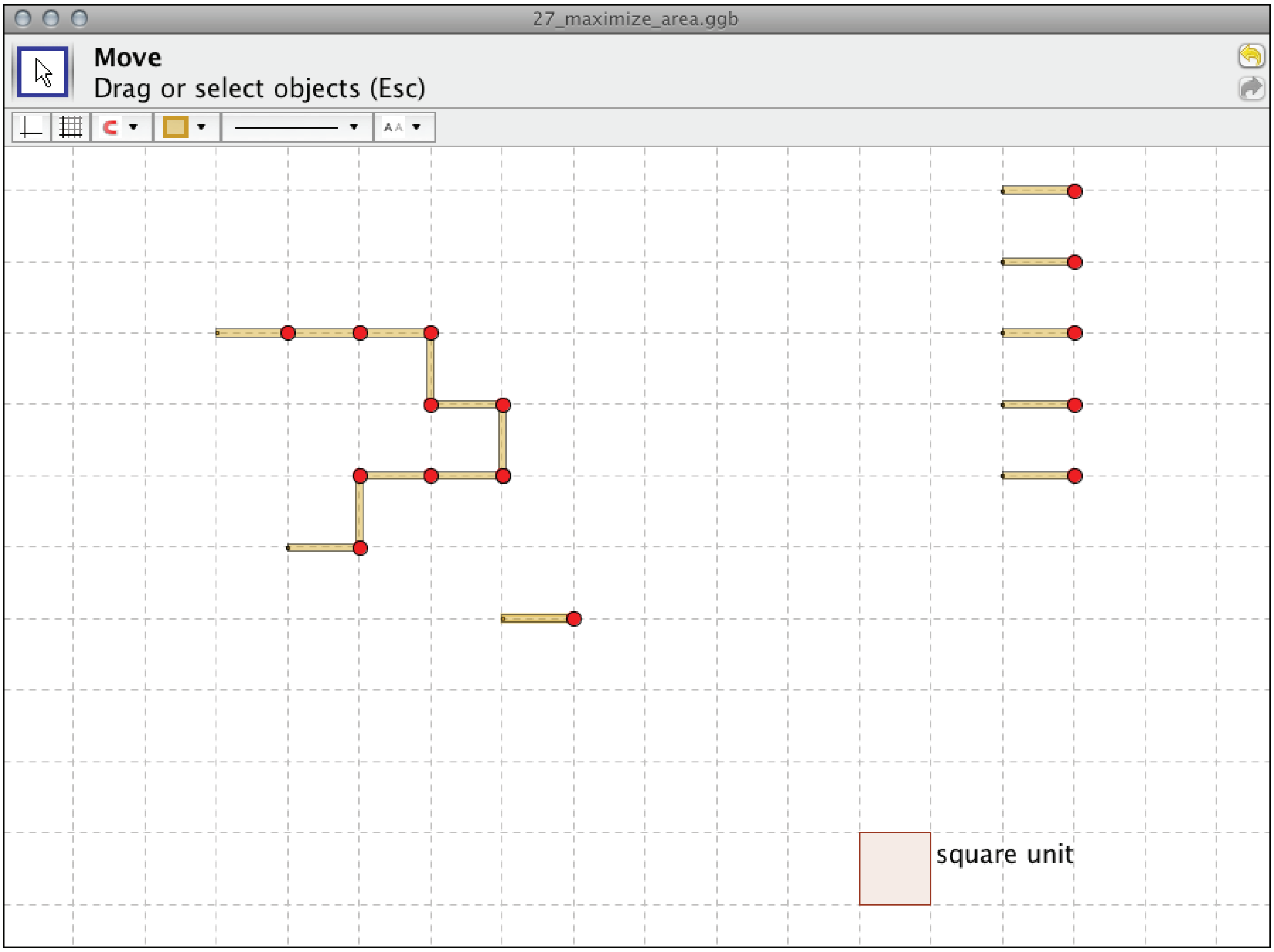


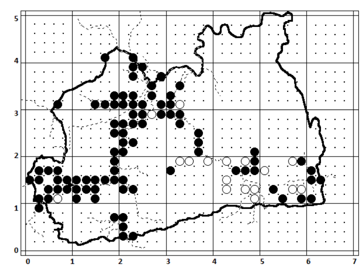
Figure 8: *Maximize the Area* GeoGebra sketch

Level: 5
Folder: coordinate system
File: 4_practice_paired_numbers_B.ggb

PRACTICE USING PAIRED NUMBERS

Background

Forest fires are often fought using special airplanes called slurry-bombers. Slurry is a liquid that helps control and put out forest fires.



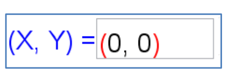
Slurry-bombers are directed to their targets by controllers using coordinate system grid maps. The controller directs the pilot to spots on the map using paired numbers. For example, "fly to and drop your load on (5,4)."

You are a controller in training. This *GeoGebra* activity has been designed to help controllers learn to use paired numbers on a coordinate system grid efficiently and accurately.

Instructions

On the *GeoGebra* page are six red target circle, fires, at various points on the coordinate system grid.

The slurry-bomber is represented by the blue symbol at the origin (0,0).



Your task is to direct the slurry-bomber to each fire six sites by entering the (X,Y) coordinates of a fire into the input box and then pressing the ENTER key.

The slurry-bomber will leave blue slurry on the points you send it to. Try to hit all the red fire circles without dropping any slurry off the targets. If you do miss a fire circle, keep going and try to hit all the fire circles with as few misses as possible.

If you have some misses and want to try again:

1. send the slurry-bomber back to the orgin (0,0)
2. pull down the VIEW menu
3. select REFRESH VIEWS

Figure 9: *Practice Using Paired Numbers* Student Handout

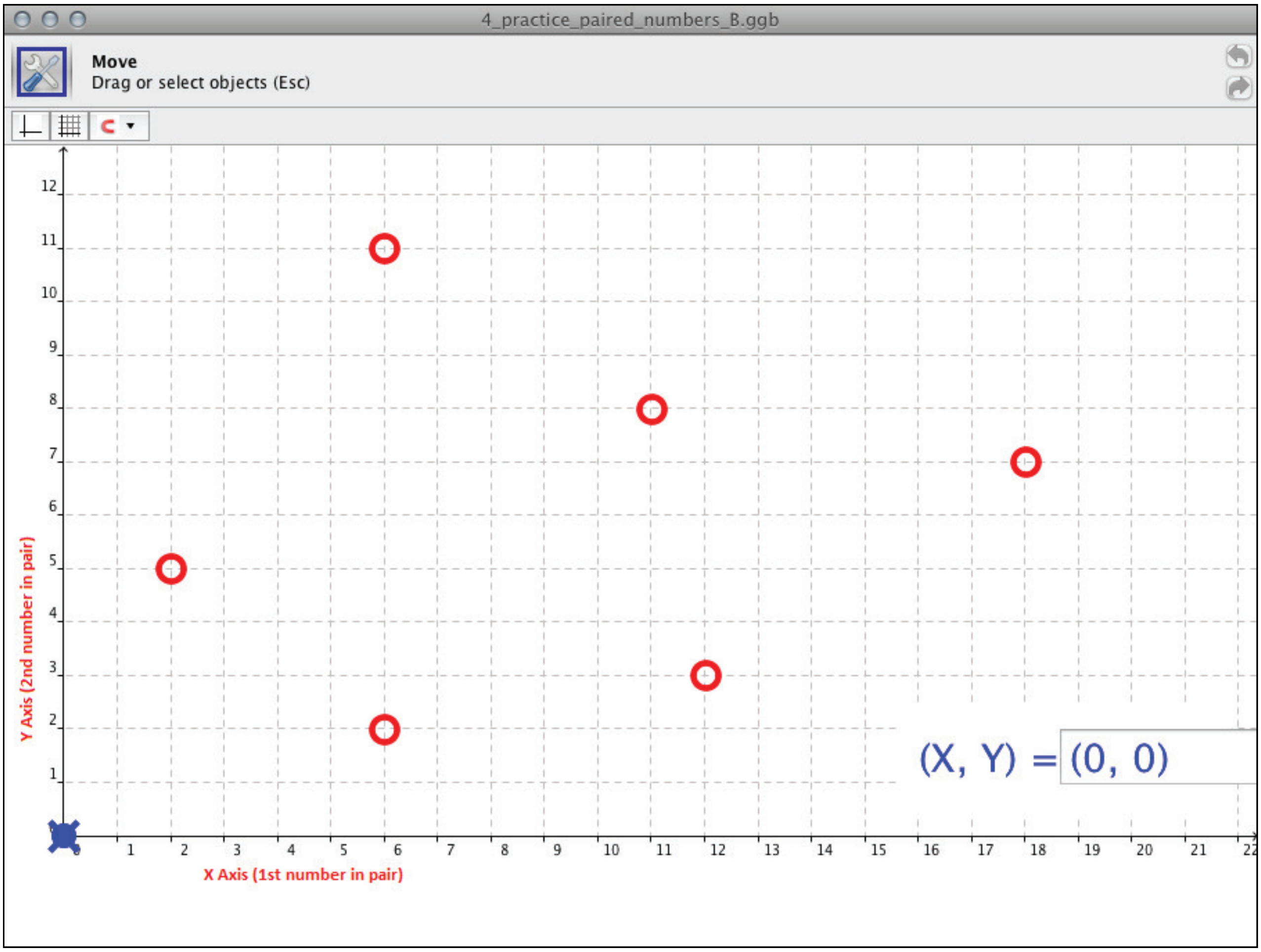


Figure 10: *Practice Using Paired Numbers* GeoGebra sketch

3.6. Standards for Mathematical Practice: Teach so that students look for and make use of structure

> Mathematically proficient students look closely to discern a pattern or structure (NGA Center and CCSSO, 2010).

The lab activities designed to develop understanding of a new concept use an inductive, laboratory approach to teaching and learning: create and manipulate geometric constructions, make observations and collect data, and then analyze the data for a pattern or structure. This technique can be used to discover both procedures and concepts.

Figures 11 and 12 highlight two student activities that use the lab approach to help students develop procedural and conceptual understanding. GeoGebra sketches designed to accompany the activities are provided at the *A Laboratory Guide for Elementary Geometry using GeoGebra* website (a url is listed on the first page of this paper).

Highlighted in Figure 11, *Area of Rectangles* is designed to address Common Core Grade 3 Geometric Measurement. The *Discover Paired Numbers* activity illustrated in Figure 12 is designed to address Common Core Grade 5 Geometry listed below.

> *Common Core Grade 3 Geometric Measurement*: Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
>
> *Common Core Grade 5 Geometry*:
>
> - Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.
> - Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

4. MORE ON THE ROLE OF GEOGEBRA IN THE ELEMENTARY CLASSROOM

GeoGebra use in the classroom has been primarily limited to the middle school, high school, and college levels. There are at least three basic strategies for integrating GeoGebra into the teaching-learning process:

- Teachers use GeoGebra to create “dynamic worksheets” for individual instruction. GeoGebra provides teachers with a tool for creating personalized interactive instructional materials, so called dynamic worksheets, by exporting dynamic figures into web pages.
- Interactive GeoGebra-based presentations, whether in dynamic worksheet format or simply standard GeoGebra files, can be used for small or large group instruction via the interactive whiteboard technologies now found commonly in elementary schools.
- Students can be taught to use GeoGebra to make their own geometric constructions as they experiment and problem solve with concepts, procedures, and mathematics related to geometry. The various GeoGebra geometric construction tools can themselves become instructional tools, for example, experimenting with the Parallel Line tool to discover the meaning of “parallel.”

All of these strategies for using GeoGebra are represented in the range of geometry activities included in the laboratory guide. Though all can be useful depending upon specific learning objectives, if a primary goal is to develop students’ skills with the GeoGebra software that can be extended throughout middle and high school, then having elementary students not only interact with teacher-developed dynamic worksheets but use the basic GeoGebra geometric construction and measurement tools to explore concepts and solve problems should be a priority.

It is worth noting that the methods, techniques, and educational philosophy influencing this perspective are not new. Early in the history of educational computing, Judah Schwartz at Harvard University not only developed a unique commercial software product for studying geometry, The Geometric Supposer, but he also promoted a philosophy of learning geometry that centered on conceptual understanding through a pedagogy based on exploration and guided-inquiry through geometric construction. GeoGebra offers an easy-to-use, free tool for teachers interested in carrying on Schwartz’s vision of learning mathematics as exploration and experience as opposed to learning mathematics as memorization without deep understanding.

REFERENCES

National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO).(2010). *Common Core State Standards for Mathematics*. Retrieved from `http://www.corestandards.org/assets/CCSSI_Math\%20Standards.pdf`

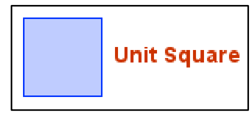
Schwartz, J.L., Yerushalmy, M., & Wilson, B. (Eds.). (1993). *The Geometric Supposer: What Is It a Case Of?*. Hillsdale, NJ.: Lawrence Erlbaum Associates.

Level: 3
Folder: Area and Perimeter
File: 4_discover area rectangle

COMPUTING THE AREA OF RECTANGLES

Introduction and Background

The area of a rectangle can determined by finding out how many unit squares it takes to cover the rectangle. But what if we don't want to simply sit and count using a grid or by moving a unit square around on a rectangle? Is there another way?



For this activity, you are going to use the Unit Square to measure both area and length. The length of any side of the unit square is equal to 1 unit.

Step 1. Construct Differing Rectangles

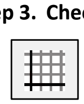
Use the MOVE (pointer) tool to drag the corner points of each square, changing them into six rectangles with *different* heights and widths.

Step 2. Determine Area Using the Unit Square

Use the MOVE (pointer) tool to drag the Unit Square to each of your rectangles and use it to determine the width, the height, and area of each rectangle. Record your data in the Results Table.

Results Table				
Figure	**Width in Units**	**Height in Units**	**Area in Unit Squares**	**Check**
Rectangle a				
Rectangle b				
Rectangle c				
Rectangle d				
Rectangle e				
Rectangle f				

Step 3. Check Your Work



Turn on and use the grid to check your results. If your area was correct, write "ok" in the **Check** column of the table. If you made an error, write the correct area in the **Check** column.

Step 4. Search for a Pattern

Look for a pattern in the areas. Do you see a way to use arithmetic to compute the area of a rectangle without moving a unit square around or counting unit squares on a grid? Describe your method of determining the area of these rectangles using arithmetic:

Figure 11: *Area of Rectangles* Student Handout

Level: 5
Folder: coordinate system
File: 1_discover_paired_numbers.ggb

DISCOVER PAIRED NUMBERS

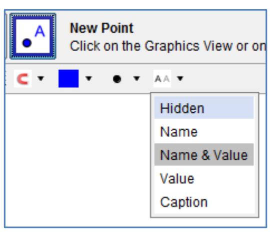
Background

If you have ever played a game of Battleship, then this GeoGebra screen should look familiar to you.

This is called a *coordinate system*. A *coordinate system* is a tool for finding exact locations on a flat surface, what geometers call a *plane*.

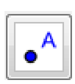
In this activity, you will experiment a bit with the coordinate system using GeoGebra, make some observations, and then make some inferences about this particular coordinate system.

Step 1. Experiment and Observe



Select the NEW POINT tool.

Before plotting any points, select Name & Value as the label for each point.



Now use the NEW POINT tool to construct a point anywhere that two grid lines intersect (cross each other).

Observe, the point you constructed is labeled “A = (number, number).”

Construct five more points and observe the labels.

Step 2. Make an Inference

Based upon your experimentation and observations, what do you think the two numbers in the parentheses mean? For example, suppose one of your points is labeled A = (2,8). What does the 2 mean? What does the 8 mean? Write your inferences here:

The first number…

The second number…

Step 3. Test Your Inference

Now construct a few more points. Were your inferences about the two numbers valid?

Figure 12: *Discover Paired Numbers* Student Handout