GeoGebra with an interactive help system generates abductive argumentation during proving process

Danh Nam Nguyen

Abstract: In this paper, the author provides a pedagogical intervention using GeoGebra in the proving process. This intervention may bridge the gap between argumentation and proof, especially the difficulty in transitioning from abductive argumentation to deductive proof. Heuristic questions and explorative tasks in an interactive help system (IHS) are responsible for sowing the seeds of realizing geometric invariants and generating the ideas for proofs. In order to explain some 'observed facts' in GeoGebra, students need to make some conjectures and then find data for validating the produced conjectures. These activities provide students with an opportunity to generate abductive argumentation aimed at writing proofs.

Keywords: GeoGebra, dynamic geometry software, help system, abductive argumentation, Toulmin model, proof, proving process

1. INTRODUCTION

A pedagogical intervention plays an important role during students' problem-solving processes. Unfortunately, this is sometimes difficult for teachers to recognize. Some researchers have concentrated on the principle of minimal help and a list of general heuristic hints (e.g. Polya, 1945; Wickelgren, 1974). These hints have been developed intuitively with open-ended and multiple-choice forms of questions which are in rapport with general principles (Trismen, 1982). Students really need these helps for assisting them in solving mathematical problems, especially in constructing a formal proof. In this research, we investigated student's understanding the development of the proving process at the tertiary level. These students took part in the "geometric transformations" course. They were allowed to use a dynamic geometry software like GeoGebra to support them in formulating and validating conjectures. Using this software, students are provided with a rich opportunity to discover the geometric problem on their own with a suitable suggestion (Hohenwarter, Preiner, & Yi, 2007). Therefore, we have classified students' levels of proving and designed an Interactive Help System (IHS) corresponding with these levels aimed at improving students' proving skills. The system includes various levels which students choose for themselves as needed. Heuristic questions and explorative tasks in the IHS with GeoGebra dynamic platform take the responsibility of visualizing geometric concepts and representing their relationships dynamically. During the proving process within a dynamic geometry environment, students need to explain some 'observed facts' such as relationships between objects, special characteristics, invariants, etc. As a result, students used abductive argumentation in order to produce supportive arguments and then reserve abductive structure of argumentation for writing a formal proof.

2. THEORETICAL FRAMEWORK

2.1. An interactive help system

The questions and tasks in the IHS should be heuristic, instructive and suitable to students' proving levels. They will appear while students interact with GeoGebra and are aimed at supporting students in constructing proofs. Before collecting empirical data, we classified students' levels of proving as follows: level 0 (information: understanding the problem); level 1 (construction: constructing the figures); level 2 (invariance: realizing geometric invariants); level 3 (conjecture: formulating conjectures); level 4 (argumentation: producing arguments); level 5 (proof: writing proofs); and level 6 (delving: delving into the problem). On the basis of these levels, we designed the IHS which provides students with ways to gain insight into the proving process.

- Information level: Support students in seizing the essence of information of the problem such as unknown, data, condition, and conclusion.
- Construction level: Suggest to students some steps

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of constructing the dynamic figures, including some auxiliary ones such as lines, segments, circles, etc.

- Invariance level: Help students search for geometric invariants by using dragging modality and then realize the invariants visually. These invariants may be constant measures, extreme value, collinearity, parallelism, orthogonality, concurrence, congruence, similarity, etc.
- Conjecture level: Guide students in formulating conjectures as much as possible, finding the data to validate correct conjectures or refute false ones.
- Argumentation level: Suggest to students to produce and collect the different kinds of arguments. During this process, abductive argumentation generates the idea for proofs, inductive argumentation checks the results, and deductive argumentation serves as a proof language.
- Proof level: Guide students in connecting and combining produced arguments in a logical way in order to shape a formal proof in the form of written structure.
- Delving level: Encourage students to reduce proof schema, delve into the problem by using mental manipulations such as generalization, expansion, specialization, analogy, decomposing, and recombining.

These levels do not appear at the same time on the GeoGebra worksheet but after a period of time. It is really needed for students to think and decide to ask for help. During the resolution process, the dialectics of conjecture and empirical findings may lead students to experience contradiction and uncertainty, opening the way to the need for explanations and overcoming the strength of empirical evidence (Hadas, Herschkotwitz & Schwarz, 2000). This strategy produced the link between experimentation and informal proof in geometry and would bring the connection to light relying on student's level of proving.

2.2. Abductive argumentation

In this research, we considered abductive argumentation as a process of producing arguments using abduction. The term "abduction" was introduced by Peirce (1960) as a model of inference used in the discovery process. It explains 'hypotheses' or 'facts' by introducing a new rule, while deduction draws necessary conclusions from the consequent of the abduction; and induction evaluates the consequent by comparing the drawn conclusions from it to experience. In mathematics, proof is deductive, but the discovering and conjecturing processes are often characterized by abductive argumentation. Therefore, this kind of argumentation can be used in analyzing a student's proof construction and supporting the transition to the proving modality as well. This means that students need to transform their abductive argumentation into a deductive one in order to construct proofs (Pedemonte, 2007). To understand how students interact with the interactive help system during the proving process, the Toulmin model of argumentation (Toulmin, 2003) was used to represent a step of an abductive argumentation although it appears as a deductive step (see Pedemonte & Reid, 2010): ?

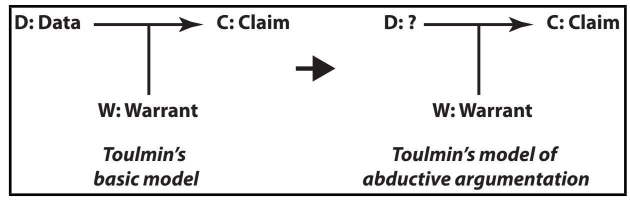

Fig 1: Toulmin's model of abductive argumentation

3. DATA ANALYSIS

During the students' proving process, the author used the screen-casting *Wink*© software to capture what and how tertiary students have done on the GeoGebra dynamic platform (McDougall & Karadag, 2008). The recording software is set to record one frame per two seconds. After gathering the data, all of the audio clips and snapshots were watched and listened to several times, so as to understand students' thinking and behaviors. The author has found that students tend to use abductive argumentation during the resolution process in order to explain 'observed facts'. It is the process of not only forming and supporting a conjecture but also generating new ideas for proofs.

The one-bridge problem below showed the way students use some open-ended questions and explorative tasks in the IHS to bring up their arguments. The conversation of three-student group has been extracted from *Wink*© audio clips and snapshots. We have also used Toulmin model of abductive argumentation to show the way students use the interactive help system to generate abductive argumentation from the invariance level to the argumentation level.

> One-bridge problem. A river has straight parallel sides and cities A and B lie on opposite sides of the river. Where should we build a bridge in order to minimize the traveling distance between A and B (a bridge, of course, must be perpendicular to the sides of the river)?

The following three-student group conversation describes how they use the interactive help system to generate abductive argumentation during the proving process.

Student 1: We can measure the length of the sum (AD + DE + EB), drag point G and observe until this sum is minimal?

Student 3: I think when point D moves to this position, the sum is minimal, you can see the minimal point of the parabola. Now we suppose that G and H are two points where we can build a bridge.

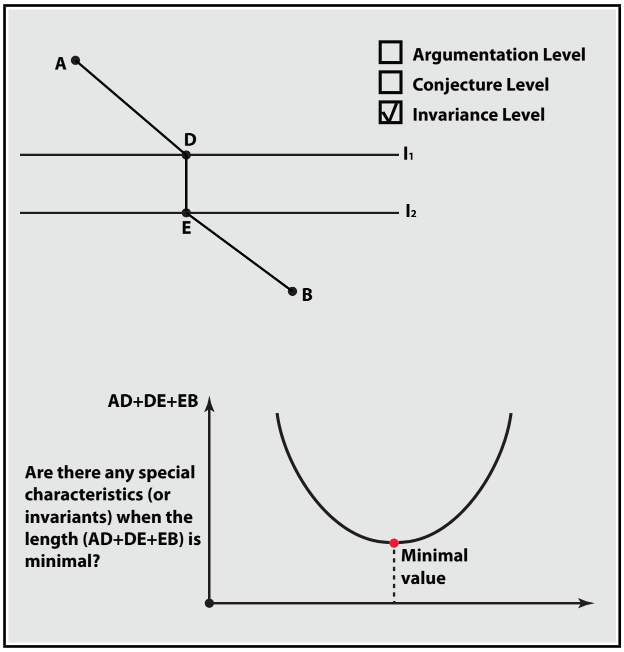

Fig 2: GeoGebra with an interactive help system (invariance level)

Student 2: Let me see. But what are special characteristics in this case?
Student 3: For me, it is difficult to see anything!
Student 1: Yeah, perhaps two lines are parallel? Look at the figure again!
Student 3: We can check it by moving point A (or point B) to the new positions and repeat this process!
Student 2: Yes, the situation is the same! It means that when the length of broken line $AGHB$ is minimal, two straight lines AG and HB are always parallel.
Student 1: That's right!
Student 2: But it is more important now, what kind of geometric transformations we should use to solve this problem based on these recognized invariants?
Student 1: The line AG can be an image of the line HB under a translation!
Student 3: The line l_1 is an image of the line l_2 under the translation of vector?

Students clicked on the button of invariance level in the interactive help system. They dragged point D on the line l_1 and observed the parabola until the sum is minimal.

Invariance level. Are there any special characteristics (or invariants) when the length $AD+DE+EB$ is minimal?

This group could not realize geometric invariants and was also not sure about invariants. They decided to click on the next button in the help system.

Conjecture level. What is the relationship between two straight lines AG and HB when the length is minimal?

Students made the first conjecture: If two lines AG and

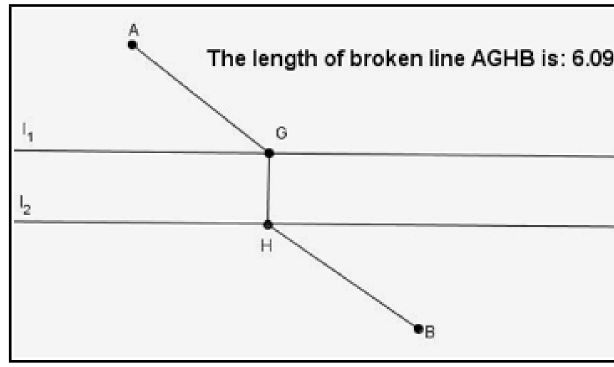

Fig 3: Conjecturing within the IHS at the invariance level

GB are parallel then the length of broken line $AGHB$ is minimal.

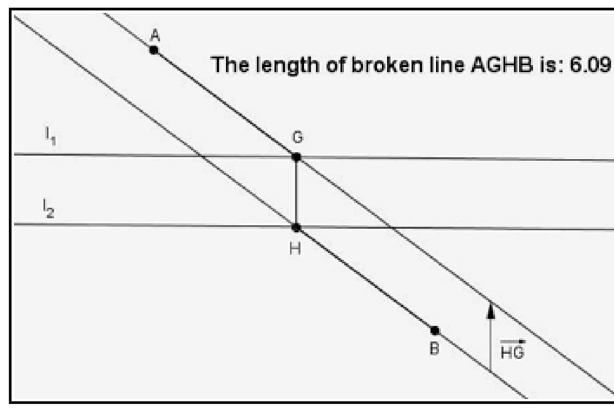

Fig 4: Conjecturing within the IHS at the conjecture level

Figure 5 illustrates this conjecture graphically using Toulmin's model of adductive argumentation.

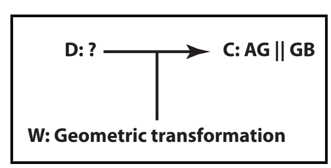

Fig 5: Student conjecture represented by Toulmin's model of adductive argumentation

Students made the second conjecture: The line AG is image of the line HB under the translation of vector $\overrightarrow{HG}$.

This group of students has used GeoGebra with the IHS to model the river and the cities. They realized that the length of the broken line is minimal when two straight lines AG and GB are parallel. This result generates abductive argumentation because students must find the data to explain why these lines are parallel. In other words, students must validate their conjectures by using 'observed facts' and abduction. Mathematics teachers should design such activities to encourage students in realizing geometric invariants and formulating conjectures.

Student 3: It is clear that the length of the broken line $AGHB$ is smaller than the length of broken line $ADEB$ but how can we check and validate this conjecture?
Student 3: We will start from the following inequality: $AG+GH+HB \leq AD+DE+EB$ But how can we prove this inequality?
Student 2: We will use the following collected data: $DE = GH = BB, DB = EB$ and $GB = HB$, hence: $AG+GH+HB = AG+GB+BB$; $AD+DE+EB = AD+DB+BB$.
Student 1: Look! We have BB in each equation, so we need only prove that: $AG+GB = ABAD+DB$.
Student 3: That is great! This inequality is triangle inequality of $\triangle AGB$!

Argumentation level. Compare the length of the broken line $ADEB$ and the broken line $AGHB$.

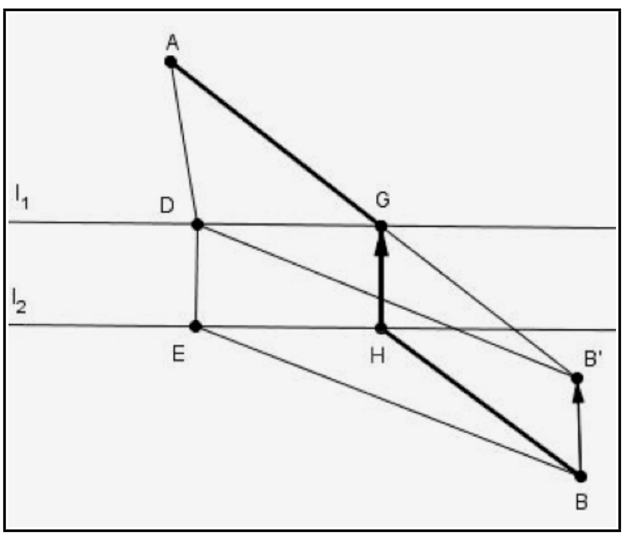

Fig 6: Conjecturing within the IHS at the argumentation level

Figures 7 and 8 illustrate conjectures generated at the argumentation level using Toulmin's model of adductive argumentation.

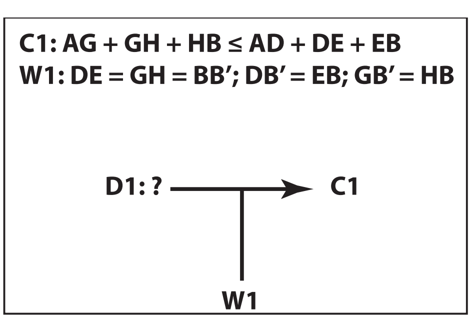

Fig 7: Student conjecture represented by Toulmin's model of adductive argumentation

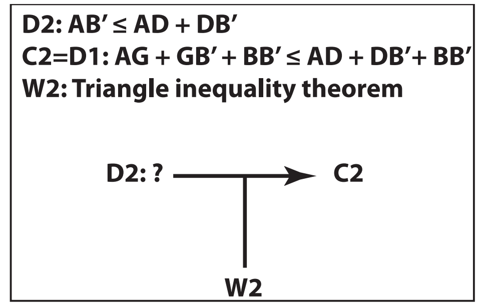

Fig 8: Student conjecture represented by Toulmin's model of adductive argumentation

Similarly, in the argumentation level, students must also explain why they attained the following inequalities by a chain of abductive argumentation and some helpful arguments for proof was also produced through this explanation. In other words, teachers should provide students with an opportunity to 'say what you see and write what you imagine'. This strategy also makes a contribution to develop the students' argumentation. In the one-bridge problem, for example, students produced the following chain of arguments: $AG+GH+HB \leq AD+DE+EB \Leftarrow AG+GB+BB \leq AD+DB+BB \Leftarrow AB \leq AD+DB$

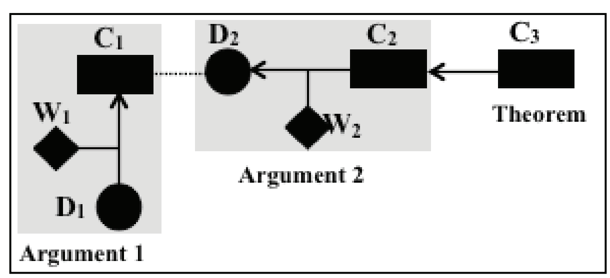

Fig 9: Student's abductive structure of argumentation

In order to convert this abductive structure into a deductive proof, students should start from final triangle inequality. Moreover, the use of dragging modalities allows students to experience motion dependency that can be interpreted in terms of logical dependency (e.g., Mariotti, 2000). Therefore, a dynamic environment, such as GeoGebra, provides the students with a rich opportunity to explore and generate informal arguments. In some cases, the students discovered accurate invariants (see Nguyen, 2011) and could validate their conjectures but they could not combine collected arguments in a logical way in order to write a formal proof. This obstacle stems from a structural gap between argumentation and proof. That is a reason why students could not transform *abductive argumentation* into *deductive proof* and failed to reverse the abductive structure of argumentation for writing proofs.

4. CONCLUSION

GeoGebra with the IHS has been shown to effectively support the students' proving process. Heuristic questions and explorative tasks motivate them to find geometric invariants and formulate the conjectures. Therefore, mathematics teachers should use this model to support their students in generating abductive argumentation and then writing a deductive proof. On the basis of these hierarchy levels in the IHS, the teachers can estimate the student's corresponding proving level. This approach also makes a contribution in granting students a broad span of a proving experiences in the mathematics classroom and deepening the understanding of the proving process within a dynamic geometry environment as well.

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