FAIR SHARE OF REGULAR POLYGON PIZZAS

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Abstract

In this article we show how to fair share a regular polygon pizza with p sides between n people The first cut point F on the boundary is given and, as in round pizzas, the cuts are done from the center C to the boundary of the regular polygon; the proof given is without words using a GeoGebra animation.

Keywords: proof without words, fair share, regular polygons, GeoGebra

1 QUESTION

How to fairly share a regular polygon pizza with p sides between n people? The first cut point F on the boundary is given and, as in round pizzas, the cuts are done from the center C to the boundary of the regular polygon.

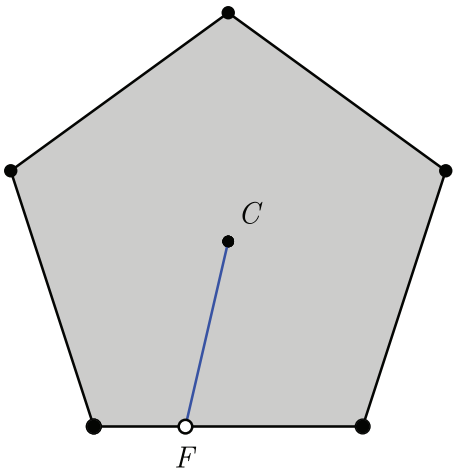

Figure 1. Regular pentagon pizza.

2 ANSWER

From the first cut F, make another $n-1$ cuts in the regular polygon so its *perimeter* is divided into n equal parts, all with the same length. That is, the other $n-1$ cut points are located along the boundary at $L/n, 2L/n, \ldots, (n-1)L/n$ from F where L is the perimeter of the regular polygon.

3 PROOF WITH FEW WORDS AND IMAGES

Each serving has the same area as it consists of triangles (or their juxtapositions) with constant altitude and constant total base length, as depicted in Figure 2[†].

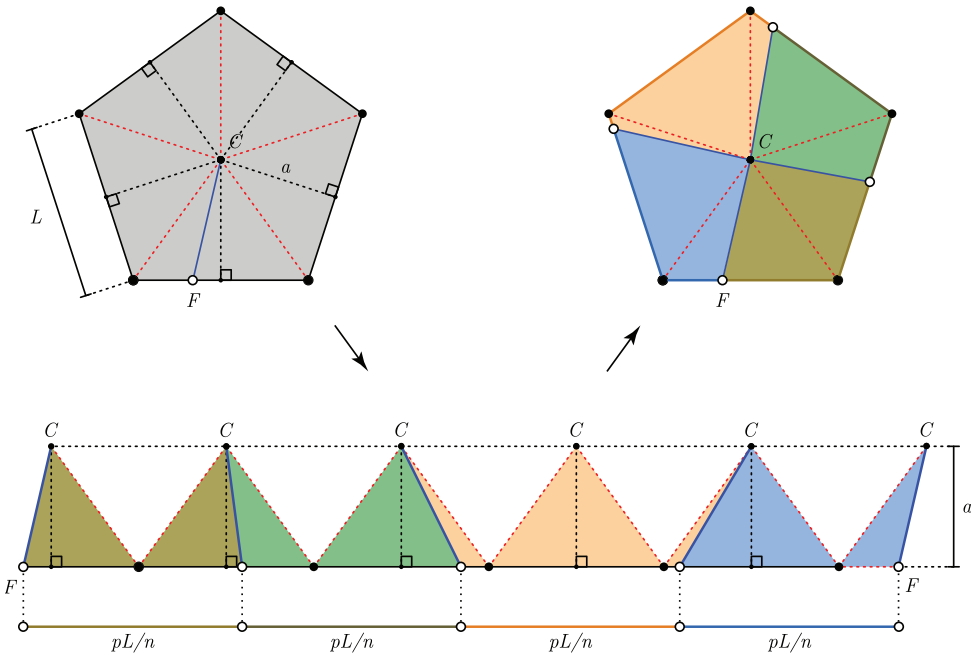

Figure 2. Dividing the perimeter in equal parts to obtain a fair share.

3.1 Conclusion

The next time you meet your friends to eat pizza, whether in the form of regular polygons or disks, do not think in terms of angles to mark the slices as it is usually done, think in terms of lengths instead. so bring a band ruler not a protactor. Other proofs without words including some involving pizzas can be found on Nelsen (1997) and Nelsen (2020).

REFERENCES

Nelsen, R. B. (1997). *Proofs without Words: Exercises in Visual Thinking*. The Mathematical Association of America.

Nelsen, R. B. (2020). *Proofs Without Words II: More Exercises in Visual Thinking*. American Mathematical Society.

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[†]An interactive applet is available at: `https://www.geogebra.org/m/nKzrxH7C`. The applet enables users to change values of p, n and the position of the first cut point F.