ON AN INEQUALITY BETWEEN SIDE LENGTHS OF TRIANGLES

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Abstract

The authors, using a proof without words style, explore a lesser known variation of the triangle inequality involving the difference between the sum of the lengths of any two sides and the length of the third side. The authors provide a web applet for further exploration.

Keywords: measurement, triangle relationships, conjecturing

INTRODUCTION

Early in their study of Euclidean Geometry, middle and secondary level students learn that for any two sides in a triangle, the sum of their lengths is greater than the length of the third side (i.e., the triangle inequality). *But what about the difference between the sum and the third side?* That is, we seek values x, y, and z where

$$x = a + b - c,$$
$$y = a + c - b,$$
$$z = b + c - a.$$

To explore these relationships, we inscribe a circle inside of a generic triangle ABC (i.e., the *incircle*), with intersection points splitting each side into two smaller segments. This is illustrated in Figure 1.

The following relationships are derived from the diagram in Figure 1.

$$AB + AC – BC = a_1 + b_1 + a_1 + c_1 –(b_1 + c_1) = 2a_1$$
$$BC + AB – AC = b_1 + c_1 + a_1 + b_1 –(a_1 + c_1) = 2b_1$$
$$AC + BC – AB = a_1 + c_1 + b_1 + c_1 –(a_1 + b_1) = 2c_1$$

From these expressions, the value of the inequality can be viewed as the sum of the lengths of the two tangents exiting from one vertex of the triangle to the circle inscribed by it, as suggested in Figure 1.

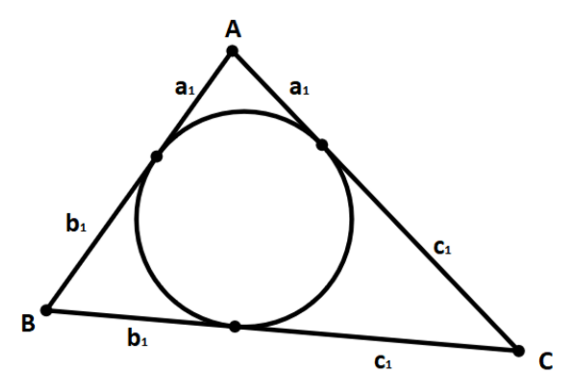

Figure 1. Incircle of triangle ABC with subsegments on each side.

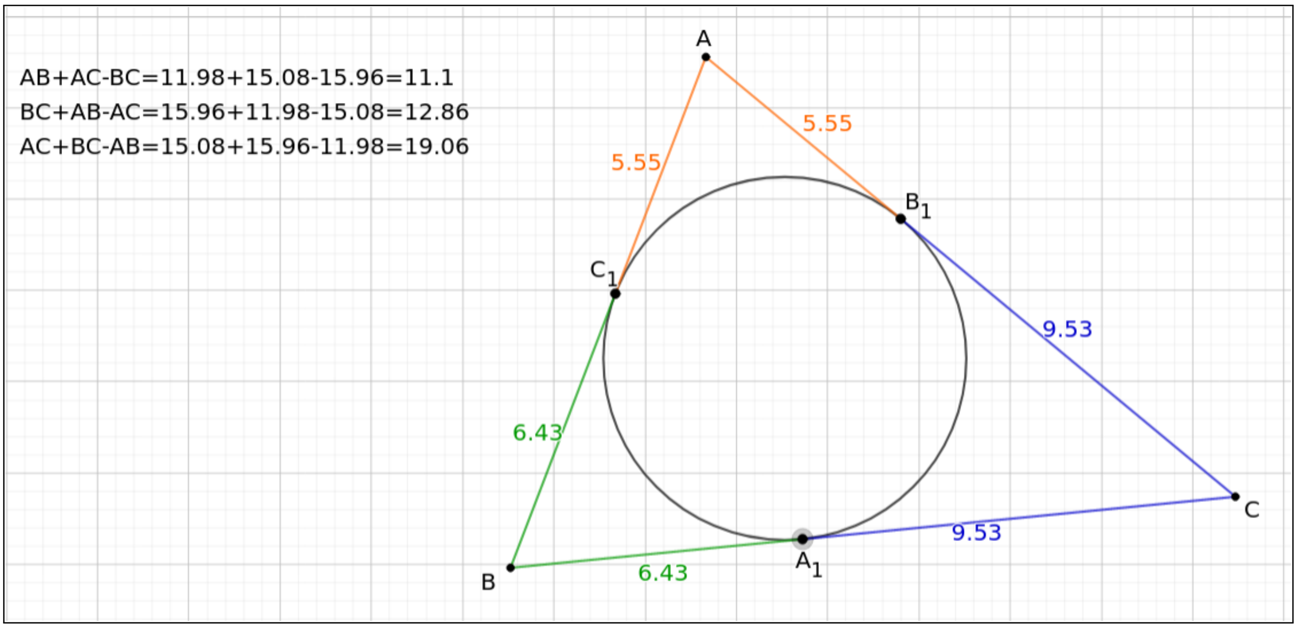

Figure 2. Dynamic applet to explore the inequality.

A Beautiful Geometric Conservation Property

To demonstrate this interpretation, we've prepared an interactive web applet that allows users to drag each of the tangent points of the triangle with the inscribed circle. Through dragging, students can change the length of the sides of the triangle, its angles, and the radius of the inscribed circle. In each case, the appropriate sizes appear on the screen that confirms the conservation property. Figure 2 illustrates basic features of the applet.

The applet is freely accessible at `https://www.geogebra.org/m/u7bfycgv`. Readers are encouraged to download the sketch, enhance its functionality, and share your results with our readers.

Dynamic Research

With dynamic mathematics software such as GeoGebra, research into relationships among the side lengths of triangles is accessible to students in the middle and secondary grades. GeoGebra makes it possible for students to drag points look for relationships among objects and measurements on the screen. GeoGebra objects can also be exported in various formats for use in other documents (Quinlan, 2013). For the many advantages and uses of computer technology in mathematical research, especially geometry, see (Stupel and Ben-Chaim, 2017; Stupel et al., 2019; Wassie and Zergaw, 2018; Zulnaidi et al., 2020). The applet that we've provided in this article will help your students conduct their own research. We believe that engagement with mathematics-based technologies fosters resiliency and grit within our students, enhancing their capacity to cope with novel tasks and unfamiliar contexts.

References

Quinlan, J. (2013). Geogebra as a frontend to generating graphics for LaTeX. *North American GeoGebra Journal*, 2(1).

Stupel, M. and Ben-Chaim, D. (2017). Using multiple solutions to mathematical problems to develop pedagogical and mathematical thinking: A case study in a teacher education program. *Investigations in Mathematics Learning*, 9(2):86–108.

Stupel, M., Sigler, A., and Tal, I. (2019). Surprising geometrical properties–their investigation, proof and generalization. *The International Journal for Technology in Mathematics Education*, 26(4):205–213.

Wassie, Y. A. and Zergaw, G. A. (2018). Capabilities and contributions of the dynamic math software, geogebra—a review. *North American GeoGebra Journal*, 7(1).

Zulnaidi, H., Oktavika, E., and Hidayat, R. (2020). Effect of use of geogebra on achievement of high school mathematics students. *Education and Information Technologies*, 25(1):51–72.

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