AN ACTIVITY IN THE STYLE OF "PROOF WITHOUT WORDS" RELATED TO ALTITUDES IN A TRIANGLE

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1 Introduction

Mathematics educators recognize the power of posing tasks with multiple solution strategies (Smith & Stein, 1998; Star et al., 2015). When students engage in multiple problem solving approaches, they begin to see connections between various areas of mathematics. Moreover, they begin to see mathematics as a means of communication with others. Without question, solving problems in multiple ways is an essential element in teaching and learning mathematics (National Council of Teachers of Mathematics, 2000; Polya, 2004; Schoenfeld, 2014). GeoGebra can assist in designing such learning activities (Oxman & Stupel, 2018). We provide one example in the following discussion, a proof-oriented task that we pose to students in introductory geometry classrooms.

Specifically, we engage students in a study of triangle altitudes through a "proof without words"—a format that can be used or adapted when teaching various geometry topics. Our activity fosters different ways of thinking that yield unexpected, short, and beautiful solutions. Teachers are encouraged to create similar lessons based on our example and this approach.

2 Launching the Activity

Consider $\triangle ABC$ illustrated in Figure 1.

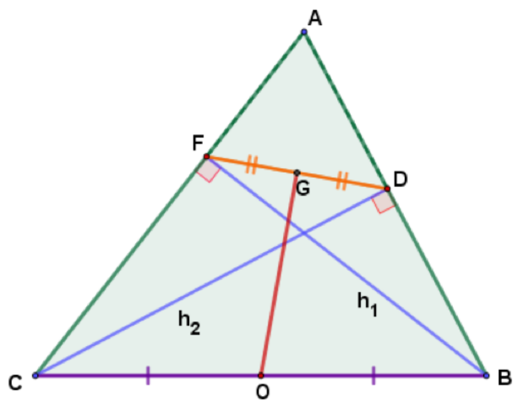

Figure 1. Students are asked to determine properties of $\angle FGO$.

Segments $\overline{BF}$ and $\overline{CD}$ are altitudes from vertices B and C, respectively. Points O and G are the midpoints of $\overline{BC}$ and $\overline{DF}$, respectively. The applet (`https://www.geogebra.org/m/bnvdggcn`) can be used to look for relationships. For instance, by dragging the vertices, it appears that $\angle FGO$ is right. In addition, look for properties and relationships among points B, D, F, and C. Feel free to click the `HINT1` and `HINT2` checkboxes included in the applet.

You might have noticed that points B, D, F, and C are *concyclic* (located on the same circle). Why is this? Take a few moments to convince yourself of this fact, then find the center of the circle. What relationship do segments $\overline{FD}$ and $\overline{OG}$ share?

In our classrooms, we ask our students to state the theorem justifying that segments $\overline{OG}$ and $\overline{FD}$ are perpendicular. We ask them to explain what happens when angles $\angle A$, $\angle B$ or $\angle C$ are right. *Do you see what happens when any of these angles are right?* Take a few moments to consider these scenarios with the GeoGebra sketch.

Students can employ multiple strategies for justifying that segments $\overline{OG}$ and $\overline{FD}$ are perpendicular. For instance, students can obtain a hint by turning off `HINT1` then clicking the `HINT2` checkbox). Students will notice that the segments $\overline{FO}, \overline{DO}, \overline{CO}$ and $\overline{BO}$ are marked as congruent (Figure 3). Teachers should prompt students to state which principle allows us to conclude that these segments are equal.

If students need more hints at this stage, ask them to justify that triangles $\triangle FGO$ and $\triangle DGO$ are congruent and, thus, angles $\angle FGO$ and $\angle DGO$ are right angles. They can also argue that segment $\overline{OG}$ is the median of the base $\overline{DF}$ of an isosceles triangle and, hence, $\overline{OG}$ is also perpendicular to segment $\overline{FD}$.

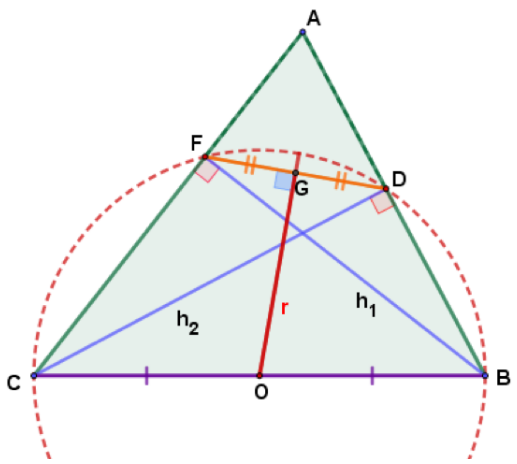

Figure 2. Points B, C, D, and F like on a circle centered at O with radius r.

The presented miniature gives an idea of the use of GeoGebra to find various solution for the same problem. Let's note the special role of the `HINT` buttons in the step-by-step progress in the search for solutions.

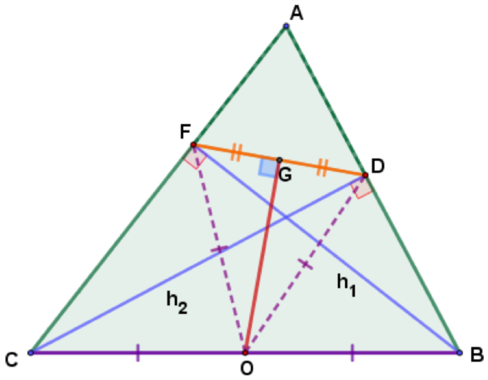

Figure 3. Construction of inner triangle $\triangle DFO$.

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