Capabilities and Contributions of the Dynamic Math Software, GeoGebra—a Review

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Abstract

In this review, the authors provide a survey of research of the dynamic mathematics software, GeoGebra, in the teaching and learning of school mathematics and related fields—including statistics, physics, chemistry and geography. The authors explore the role of GeoGebra as a tool to foster student achievement and teacher efficacy.

Keywords: meta-analysis, STEM, teaching resources, GeoGebra, technology in math education

1 Introduction

In an interconnected world, integrating technology into the teaching and learning of mathematics addresses the learning needs and interests of many of our pupils. Technology influences our students' learning styles— they prefer to see, to touch, and *experience* the topics they encounter in school. Through modelling, simulation, and visualization (Kaput et al., 2008; Ahmad et al., 2010; Guncaga & Majherová, 2012; Akcay, 2017), technology makes it possible for teachers and students to communicate traditionally abstract concepts in conceptually rich, accessible ways (Abramovich, 2013). Technology also gives opportunities for students to construct knowledge and explore new approaches to problem-solving (Bray & Tangney, 2017). Technological literacy is an essential skill that should be promoted in the mathematics classroom (Lawless & Pellegrino, 2007; Mainali & Key, 2012). Teachers need a deep understanding of the mathematical technologies at their disposal in order to fully exploit their possibilities—to design tasks that fully engage students in learning (NCTM, 2000; Kaput et al., 2008; Sherman, 2014; Stoilescu, 2009).

Historically, technological tools have played an important role in the teaching of mathematics—from abaci, writing tablets, and physical manipulatives to calculators, computers, interactive white boards, and the like (Akcay, 2017). More recently, web-based tools have become popular teaching tools. Teachers use resources such as GeoGebra, Khan Academy, IXL, NCTM Illuminations, and the National Library of Virtual Manipulatives (NLVM) to enhance their students' understanding of mathematics. Technology by itself is not the end-target of instruction; rather, the tools provide a means for students to engage in deep learning of mathematics. Technology-assisted instruction requires belief, motivation, and wise integration. Little (2008) and Lavicza (2010) discussed the barriers associated with implementing dynamic geometry software with their students. Teachers' attitudes and philosophies, the accessibility of computers, and ease of use each pose potential barriers to implementation. Researchers such as Moeller and Reitzes (2011) and Velichova (2011) have noted that integrating GeoGebra in the mathematics classroom can increase student satisfaction at all levels of instruction, while encouraging the development of skills essential for work and fostering active learning.

1.1 GeoGebra: A Brief Overview

GeoGebra is an interactive mathematics software for teaching mathematics and science, including algebra, geometry, calculus, and statistics. GeoGebra was created by Markus Hohenwarter in 2002 and has been developed by a vibrant international community. The software is multilingual in its menus and commands and has been translated into 50 different languages. In 2010, more than 5 million people visited the GeoGebra website (`https://www.geogebra.org`) from more than 180 countries (Hohenwarter & Lavicza, 2011). GeoGebra is composed of an algebra window, a graphics window (2D and 3D graphics), an input bar, and includes a built-in environment spreadsheet, CAS, and statistics and calculus tools. GeoGebra supports student engagement in mathematics at all levels.

Hohenwarter and Jones (2007) highlight the importance of GeoGebra as a tool for connecting ideas of geometry and algebra together for students. Lepmann and Albre (2008) note that GeoGebra's slider tool may be used to help students discover mathematical truths and relations while developing creative thinking skills. According to Caligaris et al. (2015), GeoGebra is practical, with a user-friendly interface and tools that enable teachers to create learning materials that range from simple graphs to dynamic web pages. The following are features that make GeoGebra a powerful teaching, learning, and curriculum authoring tool (Hohenwarter & Fuchs, 2005; Escuder & Furner, 2011; Velichova, 2011; Majerek, 2014). GeoGebra . . .

- is a freely-available, open source, multi-platform, and composed of easy-to-handle tools.
- is multilingual in its menus and its commands.
- support dynamic scenes.
- allows saving and exporting output files in multiple formats: ggb, html, xml, tikz, and more.
- allows insertion of images.
- works with LaTeX.
- has automated proof capabilities.
- has a simple user interface.
- enables multiple representations of algebraic and geometric concepts.
- enables for production of instructional materials including self-standing dynamic worksheets as interactive applets.
- has an active international community of users who provide teaching and technical support.

The authors of this paper have significant experience implementing GeoGebra in novel contexts: in MathCamp trainings and through work at a Science, Technology, Engineering and Mathematics (STEM) incubation centre for students and teachers. We've used GeoGebra with teachers and students from a wide variety of backgrounds. These experiences have motivated us to explore what others have done with the software. In the following paper, we aim to summarize both the capabilities and areas of future study of GeoGebra, synthesizing research regarding GeoGebra's capabilities and contributions to the teaching and learning of mathematics and related disciplines. Moreover, we aim to reveal possible gaps or areas of future development to the scholarly community.

2 METHODOLOGY

We've searched and selected journal articles published from 2002 to 2018 using a variety of educational databases—The Mendeley library, Science Direct library, and the Education Resources Information Center (ERIC)—and by checking the reference lists of found articles. Using keywords

“GeoGebra,” “instructional software,” and “technology in math education,” we located forty articles from the *Journal of Procedia—Social and Behavioral Sciences*, eight from the journal *Computers & Education*, approximately three hundred from the Mendeley Library, and one-hundred and five articles from twenty-five peer-reviewed journals in the ERIC database. The journals considered in the latter database included *The International Journal for Technology in Mathematics Education*, *The International Journal of Mathematics Education in Science and Technology*, *Acta Didactica Napocensia*, *The European Journal of Contemporary Education*, *Teaching Mathematics and its Applications*, *EURASIA Journal of Mathematics*, *Science and Technology Education*, and *Computers in the Schools*.

We restricted our search to peer reviewed articles, conference papers, and technical reports published in English. After locating articles, we placed them in categories: (a) Teaching and learning mathematics, (b) Teaching and learning in related fields, (c) Fostering student interest and achievement; and (d) End users’ perception about the relevance of GeoGebra. We discuss our findings in each of these four areas in subsequent sections of this paper.

3 GeoGebra in Teaching Mathematics

3.1 Calculus and beyond

A number of authors have explored the teaching and learning of specific mathematics content with GeoGebra (Hohenwarter & Fuchs, 2005; Garber & Picking, 2010; Little, 2011; Takači et al., 2015). For instance, Dikovic (2009a, 2009b) used GeoGebra applets in teaching differential calculus. Akkaya et al. (2011) describes the use of GeoGebra in the preparation of teaching materials for exploring symmetry in an analytic geometry class. Liu et al. (2011) produced virtual manipulatives to teach angle concepts, Radović (2013) discusses a virtual environment that uses GeoGebra to explore the surface areas of solid figures, Pjanic and Lidan (2015) used applets to calculate areas of trapezoids. In addition, Caligaris et al. (2015) designed GeoGebra applets to explore limits and derivatives from a geometric perspective, illustrating the relationship between the definite integral of a function and the area under a curve dynamically with sliders. In 2015, Martín-Caraballo and Tenorio-Villalón developed GeoGebra applets to work with numerical methods for nonlinear equations using several methods including bisection method, the secant method, the false-position method, and the Newton-Raphson method. Very recently, Dvir and Tabach (2017) used GeoGebra to solve extrema problems by using a non-differential approach. José et al. (2017) discusses the use of GeoGebra applets to explore the moment of inertia and to explain eigenvalues and eigenvectors from a geometric perspective.

3.2 GeoGebra as a visualization tool

GeoGebra helps students visualize functions, to determine slopes and tangent lines of curves (Garber & Picking, 2010), to understand characteristics of conic sections (Ljajko et al., 2010), and to explore geometries of objects (Budai, 2011). Students can use GeoGebra to study the basic logic of symmetry (Akkaya et al., 2011). Velichova (2011) describes engineering students modeling parametric curves—trochoidal, epitrochoid and hypocycloids—with GeoGebra. Others have discussed the study of Baravelle spirals and their connection to infinite geometric series (Escuder & Furner, 2011) and complex numbers (Gülseçen, 2012; Navetta, 2016).

3.3 GeoGebra and Proof

A number of authors have discussed GeoGebra as a tool for automated proving and justification of numerous results—from the Pythagorean Theorem, Ceva's Theorem, Thale's Theorem (Kovács et al., 2016), angle bisector theorem, side bisectors of triangles to showing properties of geometric figures like triangles and circles (Chan, 2013; Laigo et al., 2016). Moreover, others have explored GeoGebra's effectiveness as a tool for helping students solve linear optimization word problems (Molnár, 2016), in understanding plane geometry (Pereira et al., 2017), and visualizing concepts of eigenvalues and eigenvectors in linear algebra (José et al., 2017).

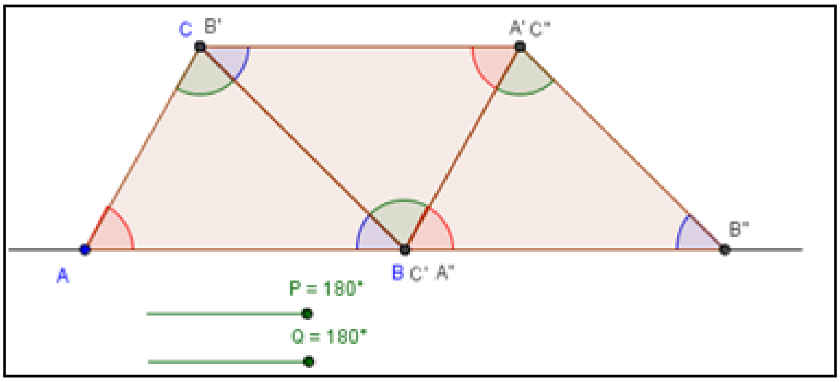


Figure 1. GeoGebra to justify angle sum theorem.

To motivate that the sum of interior angles of a triangle equals $180°$, one can use the polygon tool, the angle tool, the rotation tool, and the slider tool as suggested in Figure 1. Students use GeoGebra to explore the construction and properties of various geometric shapes—from right angled triangles and isosceles triangles to squares, rhombii, and so on. Figure 2 shows an approach for constructing a regular hexagon using the intersection points of seven circles. The construction motivates proof as students consider why the construction "works."

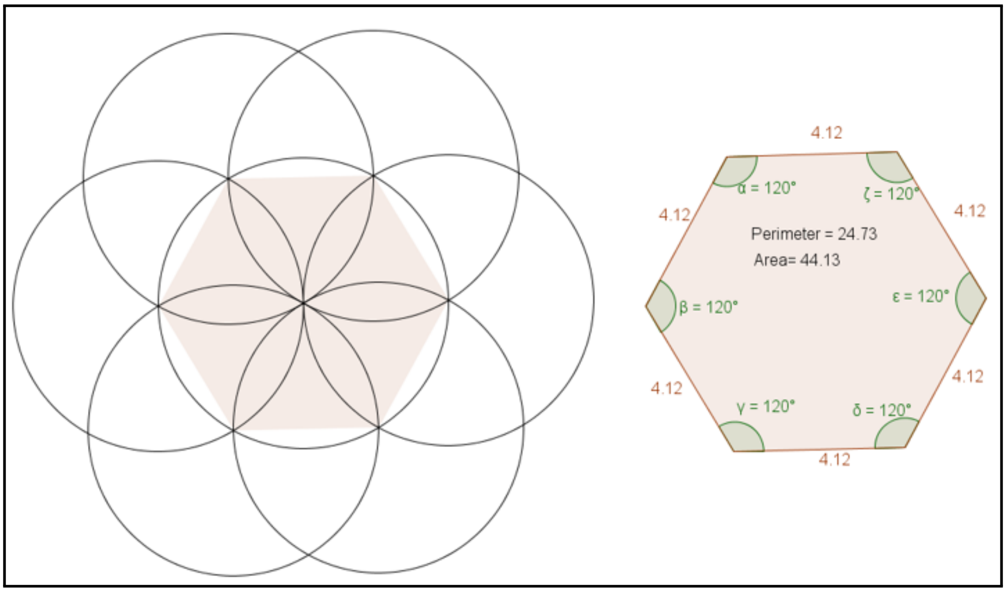


Figure 2. Constructing a regular hexagon from seven circles.

GeoGebra's capability to motivate proof of the Pythagorean Theorem is demonstrated in the studies by Stando et al. (2012) and Vargas (2013). In both studies, the authors note that GeoGebra helps to clarify the theorem, connecting it to Thale's Theorem, while encouraging student engagement. Students construct squares with the same side lengths as the triangle's sides, then they use GeoGebra's measurement capabilities to compare areas of the squares. This construction is suggested in Figure 3. Students note that the area of the square attached to the hypotenuse (c) of the right triangle equals the sum of the areas of the other two squares attached to the legs (a and b). In other words, area of square $FACG$ + area of square $ECBD$ = area of $AHIB$. Thus $a^2 + b^2 = c^2$, and the Pythagorean Theorem holds in the sketch.

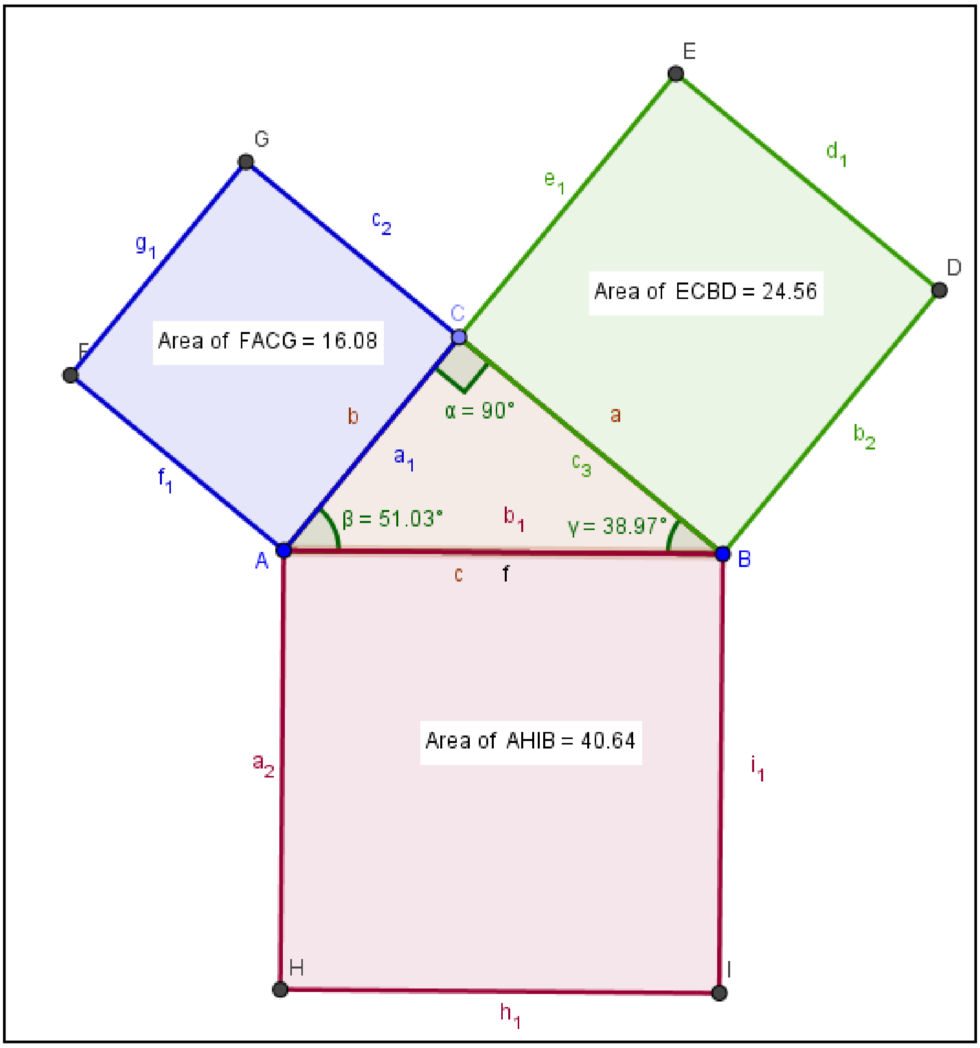


Figure 3. Pythagorean theorem using GeoGebra.

3.4 Teaching and learning transformation

The algebraic and graphic windows in GeoGebra play a vital role in visualizing rigid and non-rigid transformations: translation, rotation, reflection, and dilation. For instance, students can reflect a triangle in the first quadrant about the horizontal line, $y = 0$, and observe the effects of the reflection on the shape, area, coordinates, and side lengths. The effects of varying parameters may also be addressed (Caglayan, 2014; Anabousy et al., 2014). For instance, students use sliders to visualize the effects of the parameters a, h, and k on the graph of the quadratic function, $y = a(x-h)^2 + k$. Students begin by typing the equation $y = x^2$ in the input bar. Constructing parameters as sliders in a step-by-step method, students explore graphs of $y = ax^2$, $y = (x-h)^2$, $y = (x-h)^2 + k$ and $y = a(x-h)^2 + k$ and see the influence of each parameter (h, horizontal translation; k, vertical translation; and a, reflection and dilation). With GeoGebra students can also characterize effects of degree and leading coefficient on the graph of higher degree polynomial functions, along with the domain and range, zeros and multiplicities of zeros, maximum and minimum values, intersection points, and intervals of monotonicity.

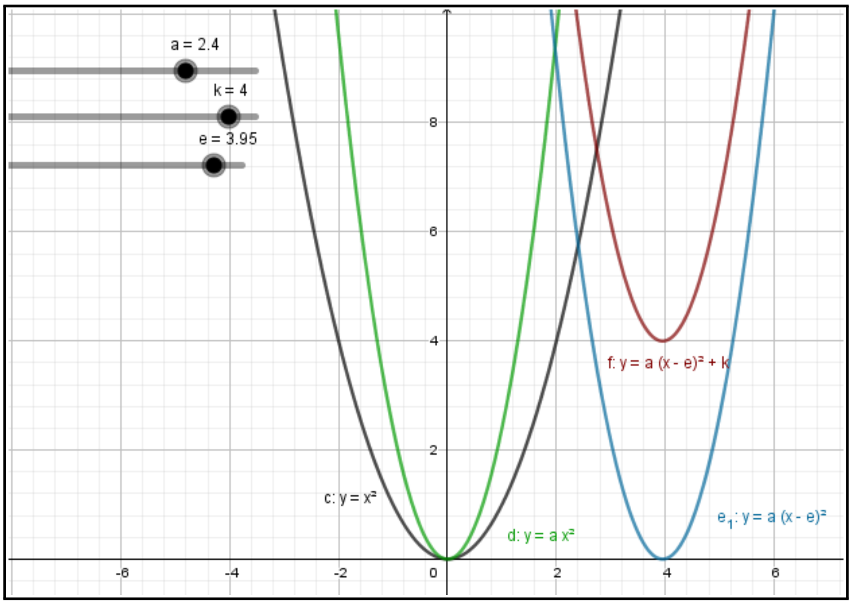


Figure 4. Effects of parameters a, h, and k on the graph of the functions $y = a(x-h)^2 + k$.

3.5 Teaching and learning calculus

The existence of freely available and multipurpose technologies provides an opportunity for teachers to reconsider the role of dynamic characteristics of calculus in its teaching. Numerous researchers (Hohenwarter & Fuchs, 2005; Hohenwarter et al., 2007; Hohenwarter et al., 2008; Little, 2011; Hutkemri, 2014; Caligaris et al., 2015; Caglayan, 2011, 2015; Takači et al., 2015; Verhoef et al., 2015; and Tatar & Zengin, 2016) have used GeoGebra to discuss a conceptual treatment of calculus at all levels.

For instance, with GeoGebra students can see how secant lines approach a tangent line at a point on a curve *dynamically*. As students drag sliders to change the width of partitions, they make dynamic comparisons of lower and upper sums with integrals. Hohenwarter et al. (2008) use examples to show how teachers can use GeoGebra to teach calculus: how to treat the concept of secant and tangent lines of functions; how to trace the slope of the trigonometric function $y = sin(x)$; how to show derivatives, roots, and extreme points; how to explain Taylor polynomials, how to explain Reimann integrals using upper and lower sums; and finally how to trace the area of a region bounded by the graph of functions. Using GeoGebra interactively, teachers and students link, *dynamically* and *visually*, the slope of tangent lines to the derivative of a function.

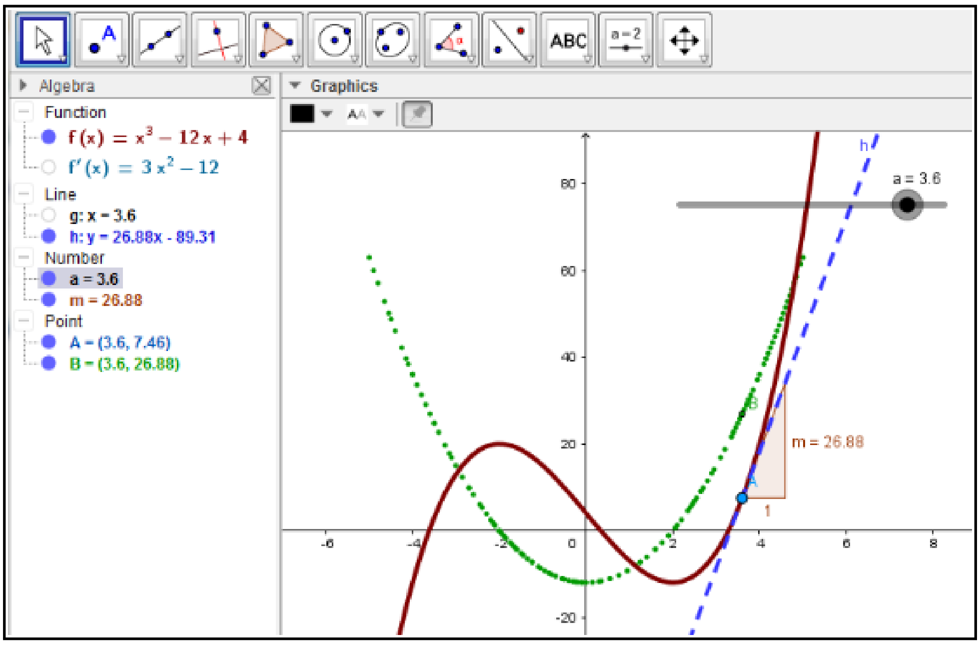


Figure 5. Derivatives of functions as a slope of a tangent line.

Figures 5 and 6 show how GeoGebra helps to illustrate the concept of derivatives and integrals, respectively. As Figure 5 suggests, the derivative a function can be determined directly from the input bar and represented symbolically in Algebra View; its graph is displayed simultaneously in Graphics View. In the same sketch, students can discover the graph of the derivative of a function from the slope of the tangent line with the help of the slider. Figure 6 suggests multiple ways that students consider integrals in GeoGebra. Note how the relationship between the upper sum, lower sum, and definite integral of a function is suggested in the sketch.

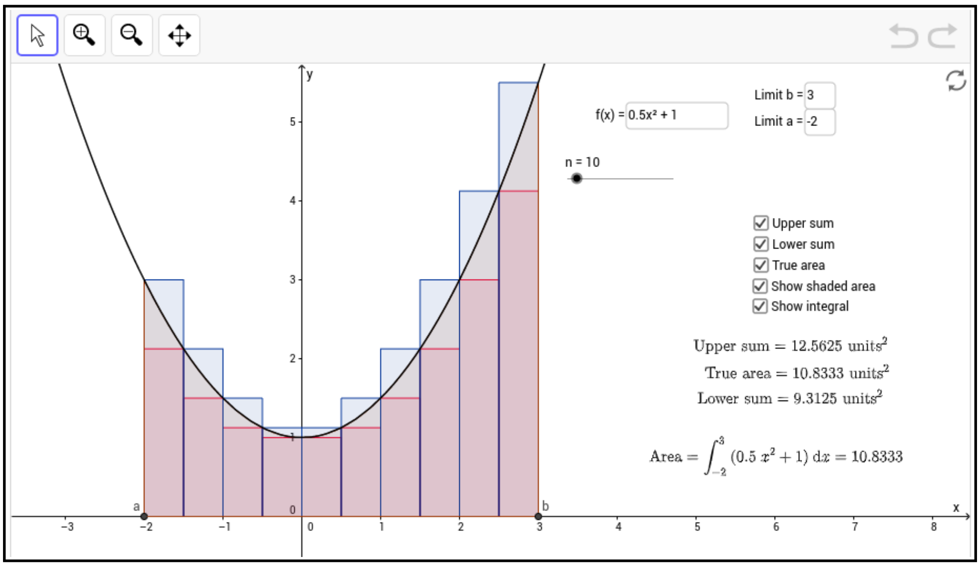


Figure 6. Lower and upper sums and definite integral demonstrated by GeoGebra.

In multi-variable calculus, GeoGebra has been used in studying properties of graphs of functions of several variables (Figure 7). Using 3D graphics capabilities of the software, students can determine domains of functions of two variables, the intersection of solid figures and a plane, contours, and so on.

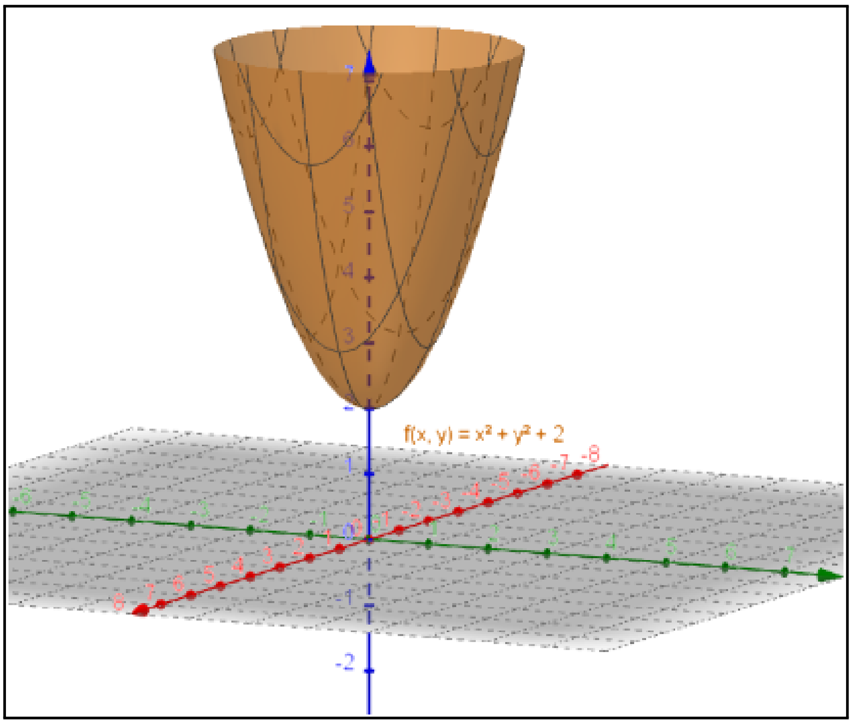


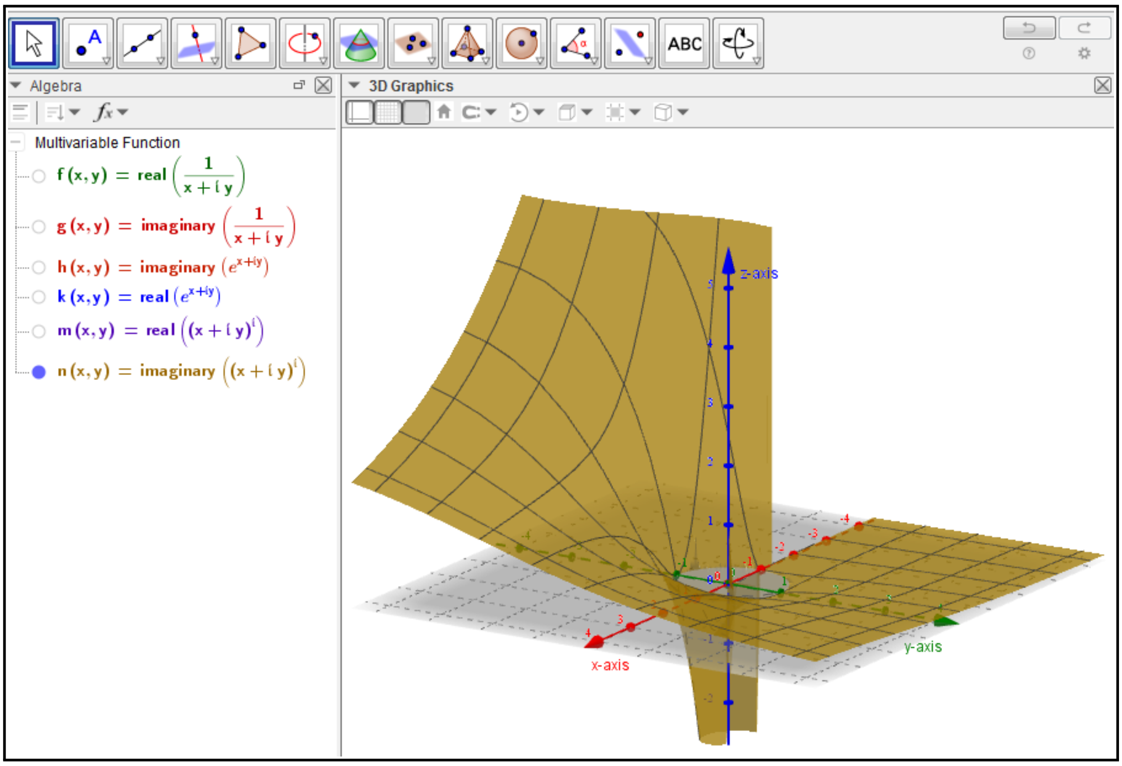
Figure 7. 3D Graphics View in GeoGebra.

Using GeoGebra to study complex variable functions is an emerging area of research. Breda & Santos (2016) have shown the potential of the software in visualizing component graphs of functions of single complex variables with domain colouring (Figure 8).

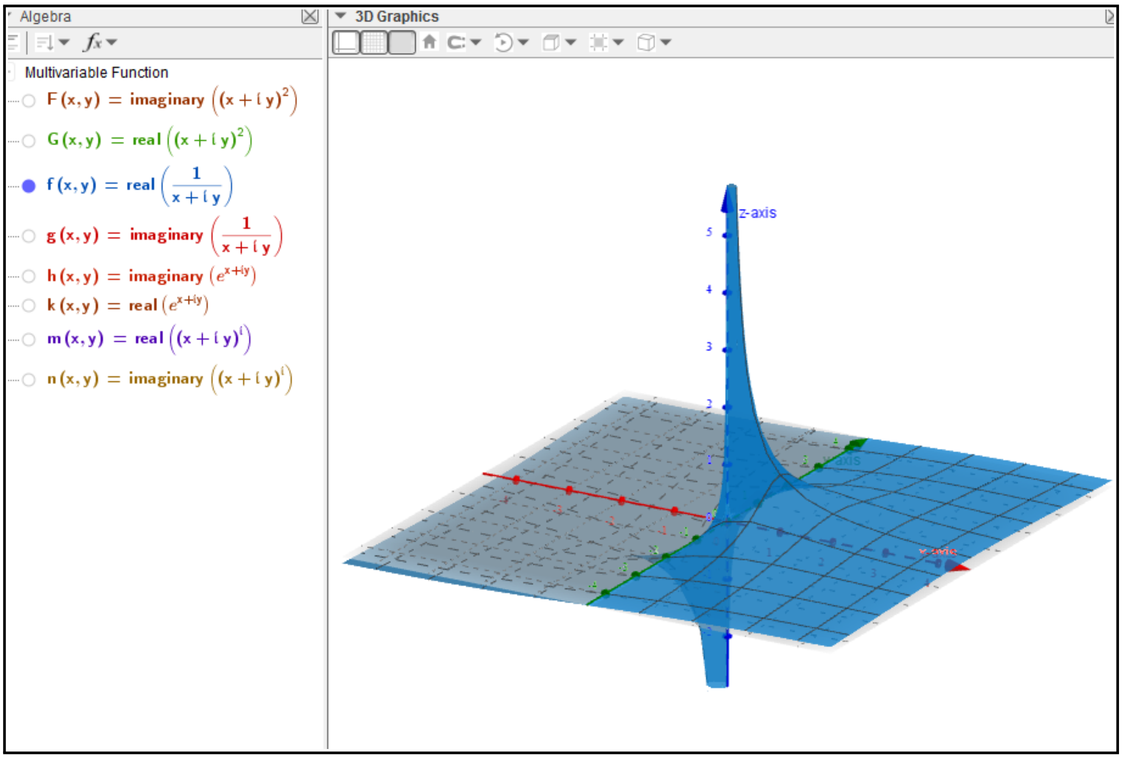
4 GeoGebra in Related Disciplines

The contribution of GeoGebra also extends to the field of statistics (Hewson, 2009; Prodromou, 2014; Phan-yamada & Man, 2018), physics (Mussoi, 2011; Malgieri et al., 2014; Yüksel & Çıldır, 2015; Walsh, 2017), chemistry (Adamec et al., 2013), geography (Soare & Antohe, 2010; Herceg et al., 2013), and mechanical engineering (Lávička & Tomiczková, 2013; Benkhellat & Bensoussan, 2017). It has played a significant role in the design of artistic graphs and games for children.

Hewson (2009) and Prodromou (2014) used GeoGebra to automatically organize data, compute measures, and generate statistical graphs. The authors specifically highlighted the importance of GeoGebra in statistical knowledge development including data management, data analysis and inference, and in exploring probability models. GeoGebra has been used in kinematics and quantum physics for the purpose of simulating natural phenomenon, to make help students more easily grasp these concepts. GeoGebra applets have been used to justify uniform rectilinear motion (Mussoi, 2011), for teaching chemical kinetics and thermochemistry (Adamec et al., 2013), for justifying mathematics and physics problems in engineering (Spyros & Nikolaos, 2013), and to represent the phenomenon



(a)



(b)

Figure 8. Graphs of (a) the imaginary part of $n(x, y) = (x + iy)^i$ and (b) the real part of function $f(x, y) = \frac{1}{x+iy}$.

of refraction at an interface (Malgieri et al., 2014). In 2015, Yüksel and Çıldır discuss the use of GeoGebra to interpret the graphs of rational and trigonometric functions in physics classes. Recently, Walsh (2017) used GeoGebra for the purpose of studying kinematics and projectile motions. Thanks to the insert image tool in GeoGebra, the multi-disciplinary role of GeoGebra has also been shown in geography (Soare & Antohe, 2010; Herceg & Herceg-Mandic, 2013). Herceg and Herceg-Mandic (2013) used GeoGebra to demonstrate map rotation, the motion of the sun, orientations in a city, and measuring the length of the road between two cities. Bézier curves, widely used in computer graphics and for animation purposes, can also be constructed with the help of the input bar and slider of GeoGebra (Figure 8). Numerous examples of interdisciplinary resources, developed by vibrant international communities, are available at the GeoGebra website (`https://www.geogebra.org`).

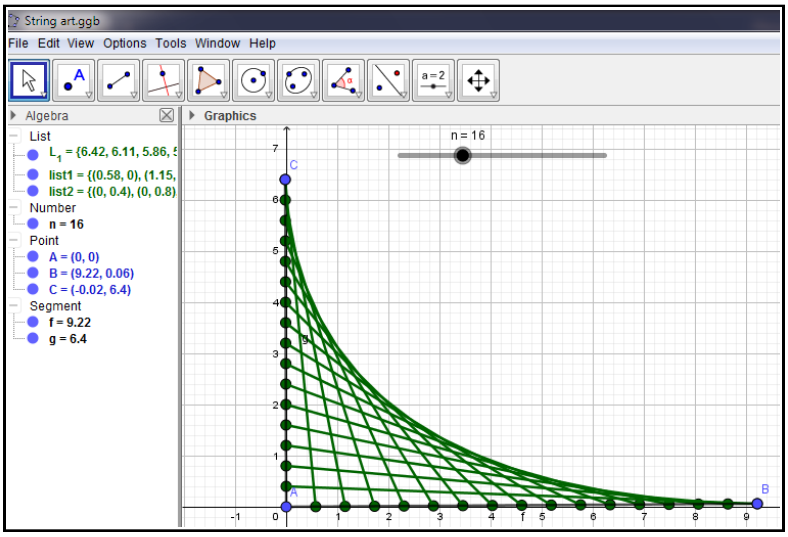


Figure 9. Bézier curves constructed with GeoGebra.

5 GeoGebra as a Tool to Foster Student Interest and Achievement

GeoGebra supplemented lessons on coordinate geometry (Saha et al., 2010; Khalil et al., 2018), on trigonometry (Zengin et al., 2012; Rahman & Puteh, 2016), in examining functions and drawing their graphs (Takači et al., 2015), on statistics (Emaikwu et al., 2015; Arbain & Shukor, 2015), in understanding theorems related to circles (Praveen & Leong, 2013; Bhagat & Chang, 2015), fractions (Bulut et al., 2016), and on transformation geometry (Xistouri & Pitta-pantazi, 2013; Seloraji & Eu, 2017) improved students' academic result. Moreover, it has been shown that students' mathematical reasoning and problem-solving skills are enhanced when the instructions are supported by GeoGebra (Acuña, 2014; Muzdalipah & Yulianto, 2015; Albaladejo et al., 2015; Granberg & Olsson, 2015; Akanmu, 2016). Students also become more motivated to study integers (Reis, 2010), parabolas (Reis & Ozdemir, 2010), angle concepts (Liu et al., 2011) and the Pythagorean Theorem (Vargas Vargas & Gamboa Araya, 2013). Students become more responsible for their own learning and actively participate more often in class when exploring topics with GeoGebra (Dikovic, 2009b; (Pierce & Stacey, 2011).

6 End Users' Perception

According to Liu et al. (2011), GeoGebra improves student interest and allows different learning styles to flourish. Students prefer learning with GeoGebra than with traditional approaches (Gülseçen, 2012; Aktumen & Bulut, 2013; Arbain & Shukor, 2015). When used as a dynamic tool, GeoGebra has the capacity to make concepts more clear, tangible, and understandable (Gittinger, 2012). Interactive graphs of different functions are easier to understand than the static ones. GeoGebra makes visualizing 3D figures easier than with paper and pencil alone. For instance, one can count the number of faces, edges and vertices of solid figures directly on GeoGebra, which is not possible on a piece of paper. Some teachers also believe that it is easier and more time-effective to teach mathematics when instructions are integrated with GeoGebra (Haciomeroglu et al., 2009; Gülseçen, 2012; Zakaria & Lee, 2012; Doruk et al., 2013; Rahman & Puteh, 2016).

GeoGebra supports both student-centered and teacher-centered approaches. Students' involvement in lessons, their collaboration, and their reasoning skills are improved while using GeoGebra (Hähkiöniemi, 2013; Granberg & Olsson, 2015; Takači et al., 2015). Jones et al. (2009) present a methodological framework that argues for GeoGebra's use both as an amplifier and an organizer. Students may use what the teacher constructed before the class, saving instruction time. On the other hand, the teacher may forward ideas so that students can investigate questions, deduce patterns, and prove conjectures on their own. In all cases, the role of the facilitator is vital and care must be given in deciding the role of the teachers, the choice of the lesson, and the design of the activities. In addition, before implementing a GeoGebra integrated lesson, teachers need to have a good sense of their students' technological fluency, they need to know how the use of GeoGebra supports their instructional goals, and they need to have a backup plan in the class in case something goes wrong with the software.

With regard to assessment, Hähkiöniemi (2013) contends that student learning is more easily assessed when lessons are supported by GeoGebra, particularly when class sizes are reasonably small. With GeoGebra, teachers can deliver immediate feedback to students or students can validate their work by themselves (Mousoulides, 2011). In fact, to facilitate the assessment, the teacher may prepare guiding practice sheets or applets accompanied by questions designed in line with the objectives of the lessons. Questions should actually trigger students to explore concepts by themselves.

7 Conclusions and Implications

In recent years, pupils have become fluent technology users, presenting an opportunity for GeoGebra to expand rapidly worldwide. Unlike other mathematics softwares, GeoGebra is a downloadable, web-based, and freely available. GeoGebra enables users to see the algebra window and the graphics window at the same time. Smart integration of GeoGebra in an appropriate classroom setting creates a flexible environment, engaging students and enhancing cooperative learning. As GeoGebra becomes more popular in a variety of countries and cultures, additional research on challenges of using GeoGebra is needed. In addition, large scale studies about possible effects of GeoGebra on students' achievement, about the necessary pedagogical knowledge required to effectively integrate GeoGebra, and challenges associated with time constraints and curricular freedom should be studied. The community needs additional articles, workshops, and conferences that explore the capabilities of GeoGebra to support mathematics education. Course curricula and textbooks, particularly those provided at teacher education institutions, should be designed in an integrated way with GeoGebra.

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