Engaging Students with Linear Functions and GeoGebra: An Action Research Study

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Abstract

Soots completed an action research study with pre-algebra students during a unit on linear functions. The goals of this study were to increase student-led technology engagement and utilize a graphing application that would allow students to make connections between multiple representations of linear functions. The use of GeoGebra combined with a student-led approach to instruction positively impacted the classroom environment, allowed students to make connections between representations of functions, and supported student discovery, especially within the topic of systems of equations.
Keywords: GeoGebra, Action Research, Systems of Equations, Linear Functions, Pre-algebra

1 Introduction

Across the country, teachers are aware of principles and standards that call for the integration of technology in the mathematics classroom. At the national level, these include the following.

- *Principles and Standards for School Mathematics: Technology Principle* (NCTM, 2000)
- Mathematical Practice #5: Use Appropriate Tools Strategically (CCSS, 2010)
- *Guiding Principles for School Mathematics: Tools and Technology* (NCTM, 2014)

Yet, there is often a disconnect between calls for technology use and implementation of technology in actual classrooms. Graduate students (teachers) in Shafer's courses report the number one reason for the lack of meaningful change is the absence of targeted professional development. Teachers lament that money is spent on hardware, software, applications, etc. with little—if any—spent training teachers to teach with the new tools. This makes sense, because most technology-focused professional development initiatives concentrate on the functionality of a tool (e.g., how to use an interactive whiteboard) or program. Little time is spent planning, implementing, and evaluating engaging activities specific to a given mathematics topic or content area. The NCTM (2014) recognizes this disconnect: "Meaningful professional development that focuses on mathematics-specific uses of tools and technology is essential to their effective use in the classroom" (p. 85). Teachers in Shafer's graduate level courses also report that classroom-based action research is an highly effective means of professional growth that increases their confidence in the use of technology and an appreciation for educational research in general. According to Mertler (2017), classroom-based action research "permits teachers to investigate their *own* practice and to discover what will and will not work for *their* students in *their* classrooms" (p. 22, emphasis in original). In what follows, classroom-based action research will be referred to as action research.

Completion of an action research study is typically required in graduate coursework. Within the structure of technology-focused action research, teachers are able to read about the benefits (and drawbacks) of a technology that can be integrated into the content they teach, locate resources to help them plan for and implement the technology in a unit of instruction, systematically collect and analyze the data, and then reflect on the results. These elements of action research provide the backdrop for Soots' experience integrating GeoGebra into a unit on linear functions with her pre-algebra students. She was aware that the effective use of GeoGebra, a dynamic algebra/geometry environment, supports the teaching and learning of mathematics with the goal of engaging students in the construction of knowledge and wanted to experience this in her own classroom. As a result, the research question she investigated is, "How can the use of GeoGebra and a student-led approach to instruction impact students' conceptual understanding of linear equations?"

1.1 Teaching and Learning Algebra

Pre-algebra is a time of crucial development for mathematics students, when they are experimenting with abstract ideas that form the foundation of algebra. Too often, students begin the study of pre-algebra struggling with arithmetic concepts like operations with integers or fractions. In many cases, students incorrectly assume that math will continue to be driven by quick mental computations, and therefore struggle to adjust to the multi-step problems found in algebra, particularly those involving functions. First, students often view tables or graphs of ordered pairs as collections of isolated points rather than one entity, making graphical analysis difficult (Knuth, 2000; Moschkovich, Schoenfeld, & Arcavi, 1993; Yerushalmy & Schwartz, 1993). Second, since students' earliest experiences with graphing involve creating a graph from a table of values, many students understand the connection from the equation to the graph, but they fail to see that the relationship works in the other direction as well. Frequently, students struggle to create equations from a given graph (Knuth, 2000). The issue becomes more complicated when students need to think about the meaning behind an entire graph as well as interpret the meaning behind a single ordered pair. This leads to a third difficulty for students; they have trouble when a situation calls for transitioning back and forth between thinking about an individual point to thinking about an entire graph and vice-versa (Moschkovich et al., 1993). Fourth, students often fail to recognize that graphs are solutions to problems in their own right, rather than merely proof for algebraic solutions (Knuth, 2000). Moreover, students struggle to interpret the meaning behind ideas such as slope or y-intercept, particularly when thinking about real-world connections. Failure to recognize connections between algebraic and graphical representations prevent students from truly understanding mathematics (Bayazit & Aksoy, 2010).

Conceptual difficulties can be discouraging in a classroom, but research indicates that these are often "transitional conceptions" made as students develop sense making related to the concepts within the study of functions (Moschkovich, 1998). "Sense making entails developing understanding of a situation, context, or concept by connecting it with existing knowledge" (Executive Summary: Focus in High School Mathematics: Reasoning and Sense Making, NCTM, 2009, para 2). Since all of these ideas are highlighted in recent state standards for 8th graders in Indiana, it has become all the more important for students to develop deeper understandings of these concepts. Furthermore, initial instruction of algebra should be grounded in the *Principles and Standards for School Mathematics* standard regarding teaching and learning linear functions in eighth grade (NCTM, 2000).

1.2 The Impact of Technology

For more than twenty years, technologies have played a significant role in fostering student conceptual understanding of function. Graphing calculators have emerged as a common tool used in algebra courses. These types of calculators offer a variety of applications in a classroom setting, including as a computational tool, transformational tool, data collection and analysis tool, visualizing tool, and checking tool (Doerr & Zangor, 2000). Regular use of graphing calculators in classrooms also impact the educational practices of teachers, often resulting in a shift from teacher as a lecturer to teacher learning alongside students (Farrell, 1996). Despite their benefits, calculators should be implemented with care because evidence suggests that struggling students' difficulties with symbolic manipulation may delay their access to the information required to use technology (Yerushalmy, 2006).

1.3 Research Rationale

Most often classroom-based action research, conducted by a single teacher completing a master's degree, does not result in the development of a novel or unique approach to teaching and learning. As a result, the overarching goal of this article is to share the journey of discovery experienced by Soots and her students within the context of an initial implementation of GeoGebra. In what follows, text written in first person is attributed to Soots.

In 2013, my district adopted one-to-one computing at the secondary level. Beginning in seventh grade, every student in the district had his or her own laptop for use in the classroom. After a full year of adjusting to this change, I began thinking about the implications of the use of laptops in the classroom. I was teaching pre-algebra using a very traditional, lecture-based method. Students made use of their laptops only to look up assignments on our learning management system, which I used as an organizational tool. With the transition to one-to-one computing, I decided it was time to incorporate student-led technology in my classroom.

Access to one-to-one computing brought several questions to mind. First, would students' understanding of linear equations benefit from an increased use of technology in the classroom? Second, if instruction of linear equations became more technology-based, what types of software or online teaching aids would be best for the students? Would incorporating the use of online graphing calculators be enough, or were other, newer technologies available to assist students in conceptual understanding of linear equations? After searching through current online graphing software, my attention was directed to GeoGebra because of its functionality, ease of use for students, and wide availability as a Google application. Completion of an action research study, conducted under the supervision of Shafer as part of a master's degree, allowed me to focus on the goal of engaging students during a unit on linear functions. As a result, I decided to investigate the use of GeoGebra in the classroom through an action research study centered on the aforementioned question, "How can the use of GeoGebra and a student-led approach to instruction impact students' conceptual understanding of linear equations?"

2 METHODOLOGY

Traditional research in education is typically conducted by researchers who are somewhat removed from the environment they are studying whereas action research in education is conducted by the teacher for themselves (Mertler, 2017). This form of research is designed to be relevant to the participants and to improve learning and teaching (Sagor, 2013). Action research differs from other types of research in that the plan of study may evolve throughout implementation as preliminary data informs the direction of continued research. Dick (2000) regards action research as a flexible methodology which incorporates both action (change, improvement) and research (understanding, knowledge), with action as the primary focus. In this study, action research was utilized because it allowed for flexibility within the proposed plan for teaching the linear equations unit. In other words, preliminary data influenced instructional choices throughout the implementation of the unit. The conclusions made through this research were limited to the experience of one class through the work of one classroom teacher, who was a novice researcher. In addition, this study was limited in that it analyzed one specific type of technology in one specific content area. The impact of the lessons in this unit was determined using a variety of data collection tools including the use of a teacher/researcher journal, student work samples evaluated through rubrics, completed activity books, and videotaped lessons. Data collection was triangulated in order to strengthen the validity of the results.

This study took place in the spring of 2015 at a junior high school located in East-Central Indiana. In 2013-2014, there were 279 students in grades 7 and 8 students enrolled, with approximately 93.5% categorized as Caucasian. Approximately 43% of the student population was categorized as low socio-economic status by the free and reduced lunch rates. In addition, the district reported a 93% graduation rate. At the time of the study, the school operated on an 8-block schedule, with students attending math class every other day for 90 minutes.

In the summer of 2015, the authors discussed co-authoring a manuscript based on the action research Soots completed. The main difficulty faced by teachers who attempt to transition an action research report, which contains well over 10,000 words and multiple tables, figures, and appendices, to a journal article involves cutting the report to a fraction of its original length. This avenue was explored with disappointing results. While the study was completed three years ago, we feel that the results are relevant as many classroom teachers face the challenge of moving to the one-to-one teaching environment. We hope that by sharing Soots' journey, others will embrace change through conducting action research on the integration of mathematics-specific technology in their own classrooms.

2.1 Procedure

I completed the action research study in a class of ten students: four girls and six boys (pseudonyms have been used). One student was above grade level in mathematics, three were below, but none of the students qualified for special education services. The unit on functions took place during a period of approximately four weeks and involved eight lessons:

- GeoGebra Basics/Function Tables
- Function Sense
- Function Notation
- Qualitative Graphs
- Slope
- Slope-Intercept Form
- Systems of Equations
- Exponential & Quadratic Functions

Because I did not have experience teaching using an interactive mathematics software program, I designed the unit with a discovery-based format. Rather than relying on lectures, lessons involved a variety of instructional strategies including some whole-class instruction, self-directed activities through discovery-based learning, group work, and interactive technology labs utilizing GeoGebra. Real-world application was included as a component throughout the unit by providing daily opportunities for students to recognize actual functional relationships.

Each lesson centered on the use of a teacher-created activity book—student "packet" of photocopied materials—containing a brief review of the previous content, instruction on new content, and a section for homework at the end. Students were asked a variety of questions within the activity book including computational, short answer, and discussion-based questions. Due to technological constraints and security issues that existed within the school network, space was provided for students to staple printed graphs that corresponded to the questions. The recent development of the GeoGebraBook provides students and teachers with an alternative to printing and stapling graphs onto a hard copy.

2.2 Lesson on Function Sense

Since the types of activities included in each lesson varied, lessons did not always follow the same format. The lesson, Function Sense, however, provides a basic framework for a typical class session in the unit. The lesson, like all others in the unit, began with a review of the previous lesson. Here, students were asked to graph the equation $4x – 2y = 3$ with GeoGebra and use the graph to identify five solutions to the equation.

The inspiration for the lesson itself came from the college textbook, *Mathematics in Action: Algebraic, Graphical, and Trigonometric Problem Solving*, which was written with a discovery-based approach (Consortium for Foundation Mathematics, 2016). The intent of this lesson was to allow students to learn about the definition of a function through discovery and observation rather than the lecture-based approach used in the past. Like the other lessons in the unit, the use of GeoGebra was woven within the examples. After the opening question, students analyzed a table which gave the number of cars in a parking lot every five minutes from 7:15 a.m. until 8:00 a.m. Students identified the input and output values in the table, which led to the definition of function. After defining the concept, there was a short discussion about why the parking lot table represented a function. Following the discussion, students analyzed a table which gave the high temperature for the first seven days in December (see Figure 1).

Daily high temperatures in Centerville, Indiana were observed and recorded for the month of December. Is the high temperature a function of the date?

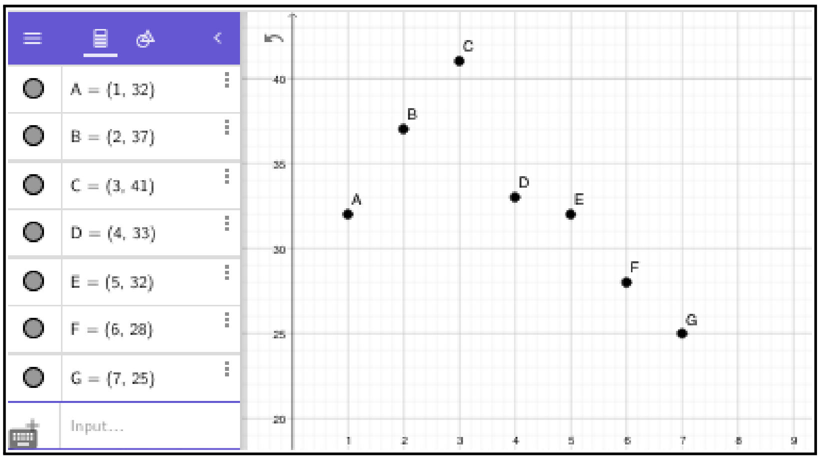
Date (Input)	1	2	3	4	5	6	7
Temperature (Output)	32	37	41	33	32	28	25

Interchange the information in the high temperature table into the table below (in other words, switch the input and output values):

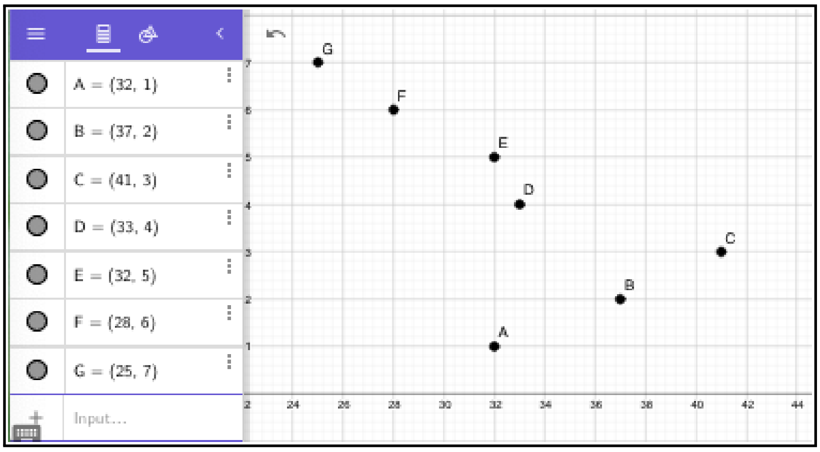
Is the new table a function? In other words, is the date a function of the high temperature?

Figure 1. Example from the student activity book.

Students determined whether this relationship represented a function. Then, students interchanged the input and output values to determine whether the new relationship represented a function. Next, students created scatter plots for the two tables in GeoGebra (see Figure 2). A class discussion followed where students discovered the vertical line test by observing the relationships between the points in the scatter plots. Students noticed that points that align vertically were the points in the table that prevented a relation from being a function.



(a)



(b)

Figure 2. Scatter plots for temperature data.

Like all lessons in the unit, homework involved the use of GeoGebra. In the homework for the Function Sense lesson, students used GeoGebra to create four scatter plots from given tables. After creating the scatter plots, students determined whether each set of points represented a function. The questions required students to explain their reasoning. For more detailed information about this lesson, refer to the full lesson plan (see Appendix A) and accompanying activity book excerpt (see Appendix B).

3 Results

Results of the action research study indicated that student conceptual understanding of linear functions was strengthened through the use of GeoGebra. In my final report (Soots, 2015), I discussed five specific instances of student growth.

First, students began the unit with lessons that taught how to use GeoGebra. During this time, I established classroom routines and provided time for experimentation, which helped to ease the use of this new technology into daily lessons. Second, students learned to use graphical solutions as an alternative to algebraic methods. This shift was motivated with directions instructing students to find solutions by analyzing graphs (discussed below). Third, students moved from thinking about a graph as a collection of isolated points to thinking about a graph holistically during the lesson on qualitative graphs. This marked a point when students began to describe the behavior of graphs using new terminology, such as increases or decreases and points of intersection. Fourth, students demonstrated conceptualization of functions and linearity by understanding the relationships of the two ideas combined through the lesson on slope-intercept form. During this lesson, students exhibited retention and a growing understanding of many concepts previously learned. Fifth, students brought new insights into their understanding of systems of equations with observations made through the use of GeoGebra.

These students' observations proved to be much more sophisticated than interpretations made by students in past years. This will be illustrated through a discussion of two lessons on systems of equations that encourage students to distinguish cases of one, zero, or infinite solutions. In the results section, I will first address how the introduction of GeoGebra affected the classroom routine followed by a discussion regarding the shift to using graphical solutions.

3.1 Classroom Routine

The unit began with lessons that taught students how to use GeoGebra. During this time, I established classroom routines and provided time for experimentation, which helped to ease the use of this new technology into daily lessons. I set aside the first day for students to learn the basics of GeoGebra. As we worked, I became a facilitator as students showed me features I did not know. One student demonstrated how to create a point on GeoGebra and drag it to join an already existing line. Another discovered that adding grid lines to GeoGebra made table creation easier. [At the time of the study, grid lines were not automatically shown.] A third student discovered that GeoGebra computes values such as $f(2)$ by typing them as commands in the input box (see Figure 3). Following the introductory lesson, I noticed a cultural transformation in the class environment. Students had a voice when it came to decision-making as they became involved in the creation of classroom routines and procedures.

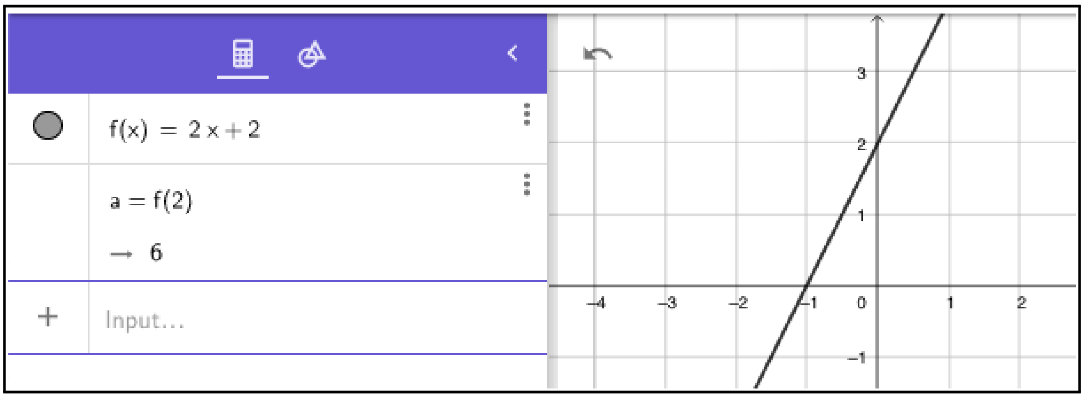


Figure 3. Algebra and graphics view for $2x + 2$.

A major issue I faced early in the unit was finding a way to accurately grade student work produced through technology. In the GeoGebra unit, I was not able to stand behind students and grade every graph they made. The print feature was not available within the application and asking students to replicate each graph by hand seemed pointless. Since my role had changed from lecturer to facilitator, I presented the dilemma to my students. They suggested that I make use of each student's Google account. Students copied and pasted their GeoGebra graphs onto a Google document titled with the day's lesson. Then, each student printed their document and stapled it to the corresponding pages in their activity book. I found that it was less time-consuming to grade hardcopies than downloading and sifting through document after document online.

3.2 Algebraic versus Graphing Methods

In algebra classrooms, students tend to solve linear equations symbolically, even in situations where the graphical solution is much easier (Knuth, 2000). Through the first half of the unit, this was the case for my students. After a lesson on function notation, students were asked to use GeoGebra to construct a table of values for the function $f(x) = 2x + 2$. Students explained that they made the graph in GeoGebra, but did not use it to complete the table. Instead, students used various input values within the expression $2x + 2$. In this case, the graph was already created, and once grid lines were displayed, finding ordered pair solutions should have been a simple task.

During the next class period, I worked with students on reading and following directions, and paying attention to words like "table," "graph," and "equation" within the directions. This seemed to help as students realized that their methods did not fit within the parameters of instruction sets. Next, I demonstrated how to find graphical solutions in each instance and how the grid lines can be used for assistance. For example, without the grid lines (see Figure 4), students had trouble identifying how many "up" and "over" each point was since it was hovering in space. The grid lines helped them know when coordinates were or were not integers. This was important, because typically 8th graders want to assume that everything is an integer.

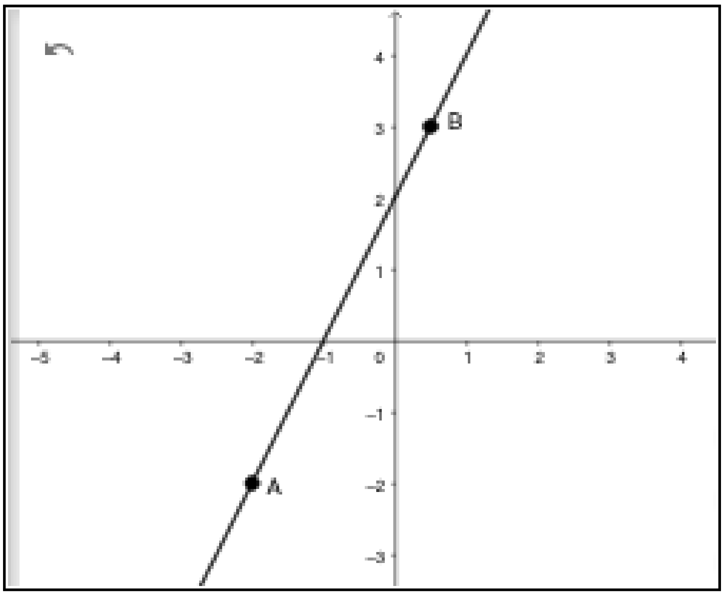


Figure 4. Function and points without grid lines.

Students seemed to catch onto this idea, and even took it one step further by using GeoGebra to add points to the line when they were checking whether they had correctly placed the "order" of the ordered pairs. Throughout these demonstrations, I could hear students say, "Ohhhh," many times, as they realized how to use the graphs in GeoGebra to answer questions. The added use of GeoGebra as a "checking tool" also helped students avoid interchanging $x-$ and $y-$values, a common mistake among students in the past (Doerr & Zangor, 2000). This shift from these students' dependence on algebraic methods to graphical methods to find solutions took place after they realized they needed to follow directions that instructed them to find a solution by analyzing a graph.

3.3 Systems of Equations

Students brought new insights into their understanding of systems of equations with observations they made through the use of GeoGebra. These observations proved to be much different than observations made by students in past years. In what follows, I discuss two lessons that took place at the end of the unit, highlighting conceptual understandings made by students through their use of technology.

The first lesson of the final section of the GeoGebra unit covered three types of systems of equations: One solution, no solution, and infinite solutions. The introductory lesson involved students working through the system $y = 2x + 1$ and $y = 3x - 5$ with substitution. For this lesson, students worked in heterogeneous groups, completing function tables for the two equations. After some thought and discussion, almost all of the students noticed that the solution to the system was the only ordered pair they observed in both tables (see Figure 5).

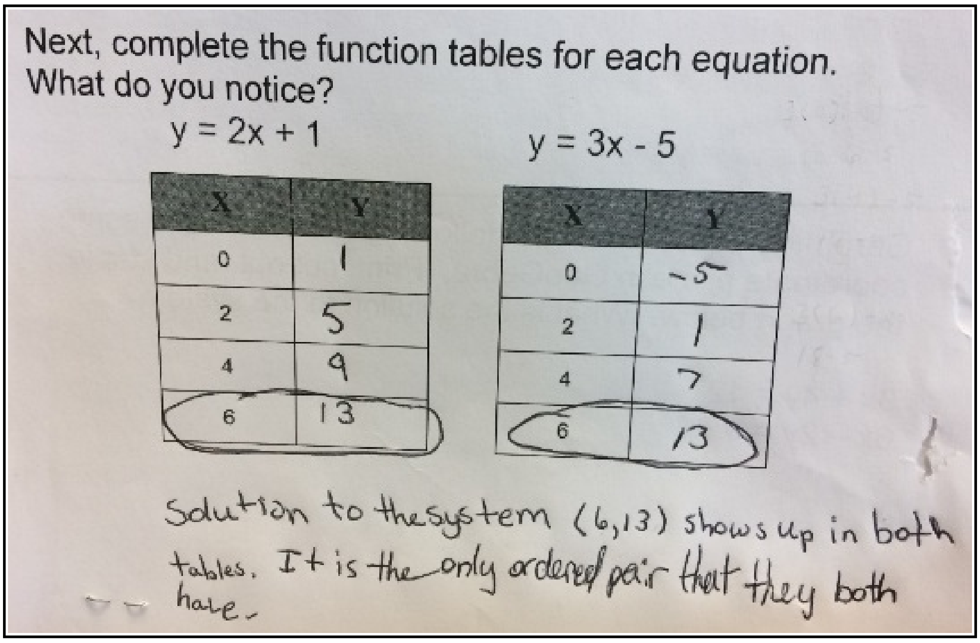


Figure 5. Ben located a single solution to the system.

Much like past years, students thought about this example and were able to conclude that this system could be solved graphically by looking for an intersection point. I was encouraged that the class realized they could find the intersection point by graphing with GeoGebra rather than applying algebraic methods. Each student tested the validity of this theory by completing the corresponding graph (see Figure 6). After seeing that their conjecture was correct, the students were ready to extend this knowledge to several more examples.

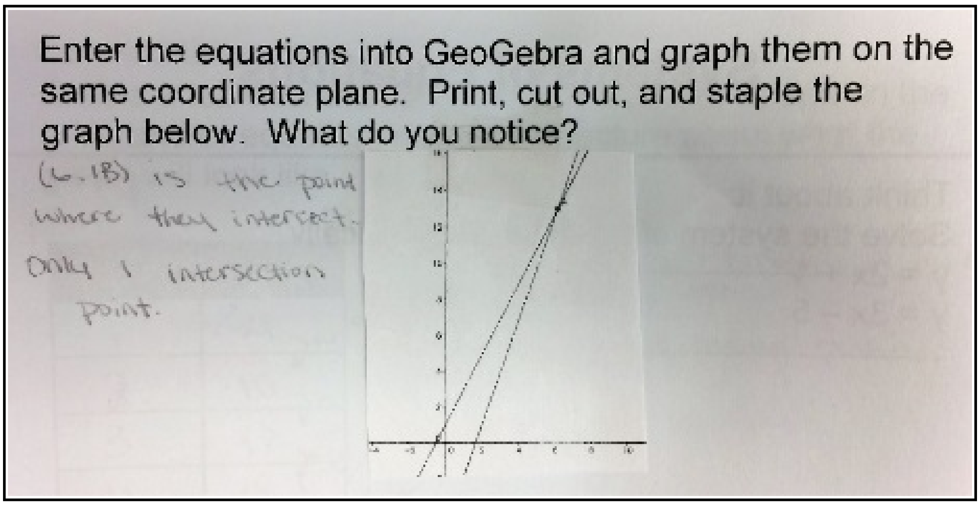


Figure 6. Ashley located the point $(6, 13)$ on the graph.

The second lesson asked the students to find the solutions to three different systems—one solution, no solution and infinite solutions—by graphing with GeoGebra. The first system had one solution and followed the introductory example discussed above. Students completed this problem individually. Ben's solution of $(2, 1.5)$, as seen in Figure 7, was a typical response.

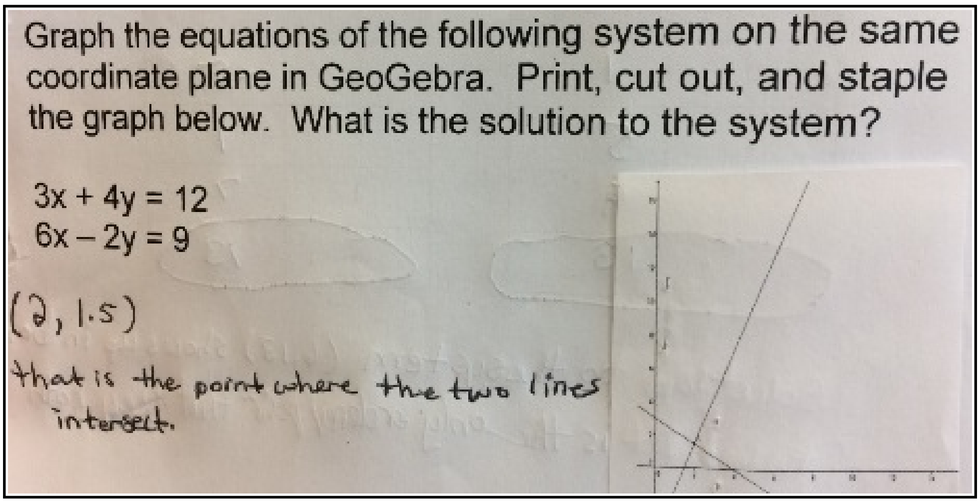


Figure 7. Ben located the point of intersection.

The second system had no solution, so this graph was different than the previous examples students had encountered throughout the GeoGebra unit. Students were surprised by the resulting graph and struggled to describe their findings. After discussion amongst themselves, students realized that the second example had to be a system with no solution, because the resulting lines had no intersection point. Hannah wrote that the system had "No Solution." She explained that "the lines are parallel. There is no intersection point" (see Figure 8).

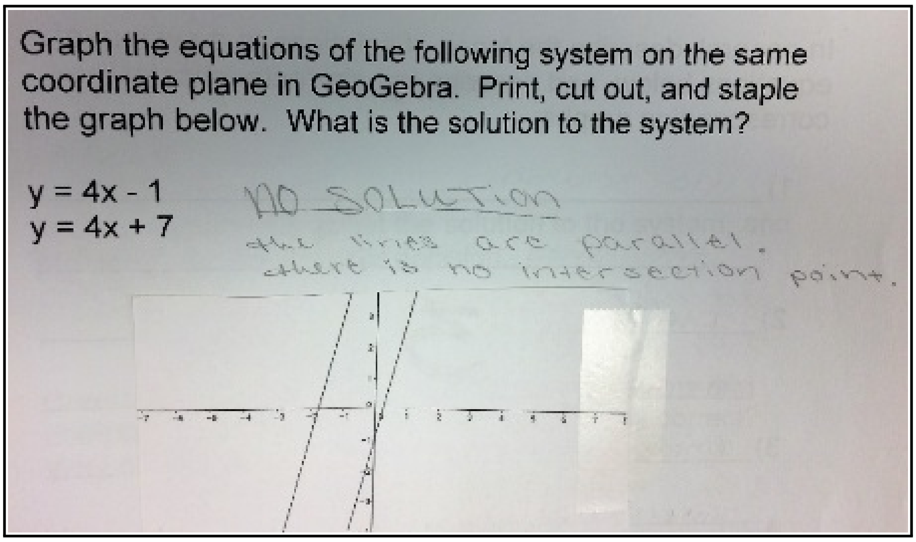


Figure 8. Hannah's written response and graph.

Difficulties continued with the third example, as this system has infinitely many solutions. Students observed that when the second equation was typed into GeoGebra, the numbers kept changing. They did not realize that GeoGebra was simplifying the equation for them, resulting in two nearly identical equations. Several students came to me, convinced that GeoGebra had some type of glitch. They were surprised when I did not respond with panic, but instead asked them to try and figure out why GeoGebra might be changing the equation. Finally, one of the students noticed that each of the terms in the second equation were divisible by two, and GeoGebra must be simplifying the equation. Hannah wrote that there were "infinitely many solutions [because] the lines cross everywhere (overlap), they are secretly the same equation twice" (see Figure 9). Hannah's creative comment demonstrates that GeoGebra enabled students to recognize the underlying pattern; equations with infinitely many solutions are identical. In Figure 10, we see that in GeoGebra the equation $-6x - 4y = -8$ was simplified to $-3x - 2y = -4$.

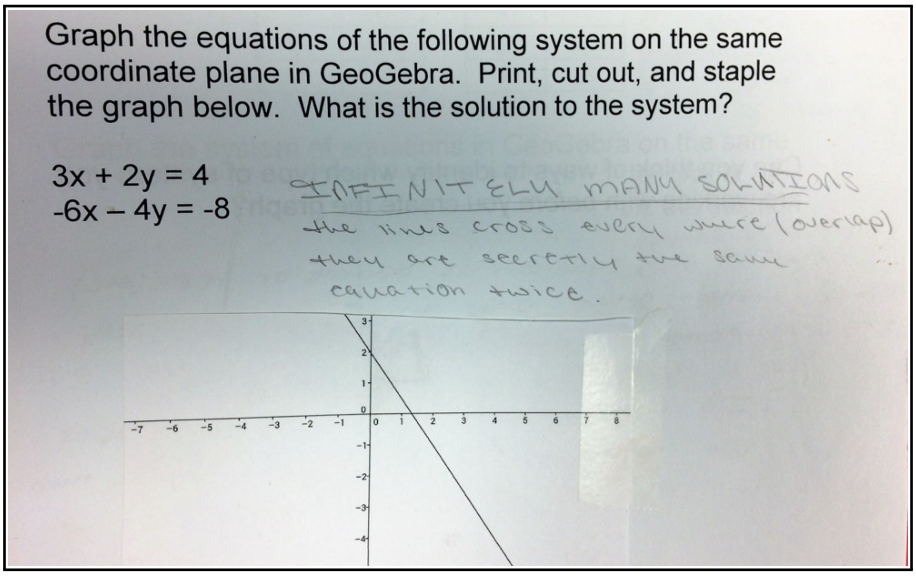


Figure 9. Hannah identified infinitely many solutions.

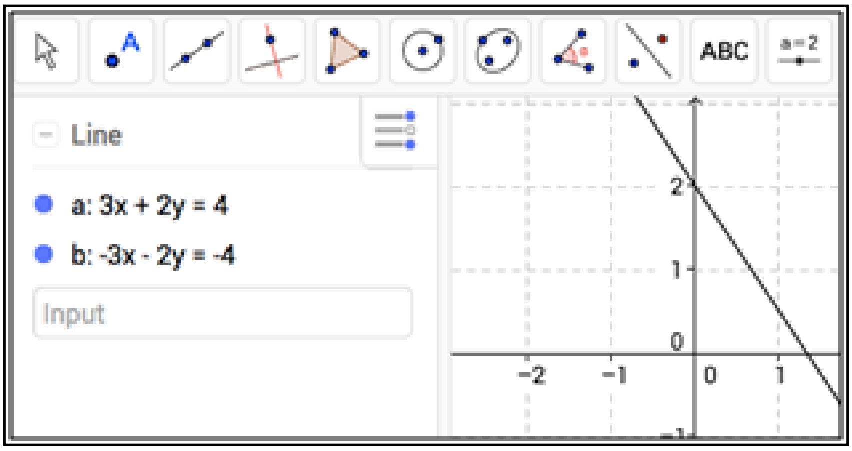


Figure 10. GeoGebra reduced the second equation.

The class discussion that resulted from these three examples was interesting. Students were able to summarize the three types of systems after creating the graphs in GeoGebra. As in the past, students realized that parallel lines would result in a system with no solution. This time, however, students recognized that there are systems with infinitely many solutions, because there can be equations resulting in overlapping lines. I asked students to think of ways they could predict the type of solution before the system is graphed. I was curious if the students could make this type of deduction. The students immediately concluded that if a system has equations that are "doubled or tripled" versions of one another, then the system has infinitely many solutions. After more discussion, students observed that the system with no solution consisted of two equations with the same slope. Ashley noted that, "if one equation is double or triple (etc.) the other one [is] infinitely many solutions. Same slope, different $y-$intersept [is] no solutions [sic]" (see Figure 11).

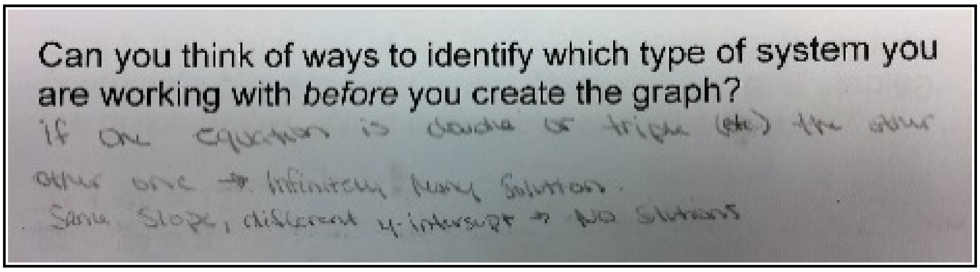


Figure 11. Ashley described her observation of doubling or tripling.

The lesson on systems of equations reinforced research by Bayazit and Aksoy (2010), who found that GeoGebra may better promote understanding between expressions and graphs because it allows for simultaneous manipulation. During the lesson, this group of students observed relationships between equations of systems with infinitely many solutions and their corresponding graphs. Because they noted that GeoGebra automatically reduced the equation when graphing, students deduced that systems with infinitely many solutions have equivalent equations. In prior years, my students had not been able to recognize this relationship without being told directly.

These questions from the final lesson: "What conclusions can you draw about the algebraic solution, the function tables, and the graphs of a system?" and "Can you think of ways to identify which type of system you are working with before you create the graph?" challenged students to make connections across these representations. This causes students to move beyond the goal of recognizing an intersection point on a graph as a solution to a system of equations required in the Indiana Academic Mathematics Standards 8.AF.8:

> Understand that solutions to a system of two linear equations correspond to points of intersection of their graphs because points of intersection satisfy both equations simultaneously. Approximate the solution of a system of equations by graphing and interpreting the reasonableness of the approximation (2014, p. 10).

4 Discussion

The implementation of one-to-one computing and the timing of graduate level coursework prompted me to plan and implement a technology-based unit. My goals were to increase student-led technology engagement and utilize a graphing application that would allow students to make connections between multiple representations of linear functions. This goal resonates with a caution found in the obstacles section of Principles to Action (2014): "However, the value of the technology depends on whether students actually engage with specific technologies and tools in ways that promote mathematical reasoning and sense making" (p. 80). The results presented in this article provide evidence that my students were truly engaged with the technology. Through the implementation of the GeoGebra unit, I provided opportunities for a variety of instructional strategies including whole-class instruction, self-directed activities through discovery-based learning, group work, and interactive labs. Carefully planned tasks and meaningful discussions allowed students to connect new learning with prior knowledge. The impact of completing action research extends beyond the development and implementation of a technology enhanced unit of instruction. In what follows, I reflect on the impact of completing my GeoGebra study.

5 Reflection

Since I began my teaching career in 2004, I attended several seminars and read many articles on technology in mathematics education. These opportunities had value but were not enough to prompt me to make changes. Because incorporating technology in my classroom involved a major shift in teaching style and classroom management techniques, I needed a bigger catalyst. In my case, completing an action research study as a capstone to my master's degree caused me to write my first technology-based unit. Making changes to my teaching practice required risk, but the flexibility of action research mitigated my fear that this new approach might not work. The rigor of action research facilitated an investigation into the literature on interactive mathematics software programs. Also, the reading I completed prior to implementing the GeoGebra unit made me think about students' "transitional conceptions." This idea was introduced in an article written by Moschkovich (1998). There are times that students appear to be making mistakes, but upon closer inspection, I realize they are demonstrating sense making that gives them partial understanding. When I teach now, I probe students more often to better understand their thought processes. Even though the article was related to errors with $x-$intercepts, I have found that this idea of partial understanding relates to every other area in mathematics. The action research helped me strengthen my teaching because it prompted me to push students through their misconceptions. In closing, this story does not end with what took place during my action research study. I continue to successfully expand my use of GeoGebra—and other technologies—to additional content areas and grade levels.

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Appendix A – Function Sense Lesson Plan

Unit: Functions
Topic: Function Sense

Indiana Academic Standards:
8.AF.3 Understand that a function assigns to each x-value (independent variable) exactly one y-value (dependent variable), and that the graph of a function is the set of ordered pairs (x, y).
PS.4 Model with mathematics.
PS.5 Use appropriate tools strategically.

Purpose:
Students will recognize a function from a table of values and/or a graph.

Anticipatory Set:
Refer to the linear equations activity book. Students will be given the equation $4x - 2y = 3$. Students will use GeoGebra to graph the equation. Students will find five solutions to the equation by analyzing the graph. Students should be instructed not to use the table feature in GeoGebra. Once students are finished, discuss correct answers as a class.

Procedure:
1. Refer to the activity book. Students will be given a table with times and the number of cars in the school parking lot. Students should answer four questions about the table. First, they should identify the input and output variables. Second, they should find the number of cars that corresponds to a time of 7:35 a.m. Third, they should find the number of cars for an input of 7:45 a.m. Fourth, they should analyze how many different outputs there are for each input in the table.
2. Define function and briefly discuss the concept with the class. Students should enter the definition in the activity book and then explain how the parking lot table is a function (in complete sentences).
3. Students will analyze a table giving dates and high temperatures. Students should determine whether high temperature is a function of the date. Then, students will interchange the information in the high temperature table into a second table to determine whether the date is a function of the high temperature. Next, students will create graphs for the two temperature tables in GeoGebra. Students should print these graphs and fix them into the activity book.
4. Begin a class discussion where students compare the information in the tables to the information in the graphs. Through the discussion, lead students to notice the vertical line test.

Closure:
Students should complete two reflection questions about their understanding of functions: “List some methods you can use to determine if the values in a table represent a function,” and “List some methods you can use to determine if a graph is a function.”

Adaptations:
The list of exercises in each section may be shortened depending on the abilities of the student.

Materials:
Linear Equations Activity Book for each student, Laptop for each student, GeoGebra app on Google, the materials to print/staple graphs

Homework:
Students should create four graphs on page 13 of the activity books. After completing each graph, students should determine whether each represents a function and provide an explanation.

Appendix B – Student Activity Book Excerpt

Function Sense

Think about it:

Use GeoGebra to graph the equation 4x – 2y = 3. Use the graph to find 5 solutions to the equation. *Do not use the table in GeoGebra!*

List your five solutions here:

The number of cars in the Centerville Jr./Sr. High parking lot was counted every five minutes from 7:15 a.m. to 8 a.m. on a Monday morning. The results are shown in the table:

Time	# Cars
7:15	6
7:20	9
7:25	22
7:30	30
7:35	41
7:40	58
7:45	78
7:50	85
7:55	97
8:00	101

In the table, identify the input variable and the output variable.

For an input of 7:35 a.m., how many cars are in the parking lot (output)?

For an input of 7:45 a.m., how many cars are in the parking lot (output)?

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For each input (time of day), how many different outputs (number of cars) are there?

The set of data in the parking lot table is an example of a mathematical function.

A ______________ is a correspondence between an input variable and an output variable that assigns one unique output value to each input value. So, any input value has only one output value.

Explain how the parking lot table fits the definition of a function:

Daily high temperatures in Centerville, Indiana were observed and recorded for the month of December. Is the high temperature a function of the date?

Date (Input)	1	2	3	4	5	6	7
Temperature (Output)	32	37	41	33	32	28	25

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Interchange the information in the high temperature table into the table below (in other words, switch the input and output values):

Is the new table a function? In other words, is the date a function of the high temperature?

Using GeoGebra, create two different graphs of the temperature tables. Print, cut out, and staple them below:

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During the class discussion, list the observations people make about the graphs they have created:

List some methods you can use to determine if the values in a table represent a function:

List some methods you can use to determine if a graph is a function:

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For next time:
Using the methods you just listed, create four graphs for the tables below using GeoGebra. Print, cut out, and staple the graphs below. Explain whether each is a function.

X	Y
1	7
-4	2
0	0
5	-1
7	4

X	Y
0	5
1	2
-4	7
1	-6
6	-1

X	Y
-2	0
0	-4
1	5
-2	5
6	-1

X	Y
0	5
-1	4
7	3
2	-1
3	4