“MESSING AROUND” IN GEOGEBRA: ONLINE INQUIRY THROUGH APPS AND GAMES

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Abstract: This article explores a problem solving strategy called “messing around” that is particularly apt for online course work in GeoGebra. While traditional math problems may require students to search deliberately for a specific solution, “messing around” employs a more fortuitous approach. We dissect this approach and examine multiple GeoGebra apps that support “messing around” by providing students with a sandbox for mathematical experimentation.

Keywords: problem solving, online inquiry, hanging out, messing around, geeking out, GeoGebra

Introduction

There is already a wealth of research exploring students’ problem-solving strategies in mathematics. Pólya [14] proposed students use a series of heuristics such as “work backwards,” “consider special cases,” and “solve a simpler problem.” These heuristics are employed through four steps: (1) understand the problem, (2) make a plan by selecting one of the heuristics, (3) carry out the plan, and (4) review your work. Most subsequent problem-solving research has simply expanded on Pólya’s heuristics, scaffolding the process in similar ways [4, 10, 13]. Schoenfeld [17] found that amateur problem-solvers tended to select one technique and stubbornly stick with it even when their approach was not working. On the other hand, advanced problem-solvers spent more time testing different strategies until they found the right one. In all of this work, the prevailing idea is to ask students to think metacognitively about the different strategies available to them so they can work more deliberately towards a clear end-goal.

Most of this research was conducted before the advent of so called new media technologies: interactive, digital media such as blogs, wikis, video games, social media, and dynamic graphing software. As such, this research does not consider how new media technologies might influence the problem-solving process. In a recent study [16], Santos-Trigo and Moreno-Armella examined how teachers and learners use new media technologies like GeoGebra to communicate about mathematics during the problem-solving process. They observed that new media can provide online resources for students to extend discussions beyond the classroom, and these technologies also offer interactive models of mathematical objects that students can use to test their understandings as they work. However, research in this direction is still very limited, and most new media research focuses on users as “digital natives” or “millennials,” attributing shifting forms of engagement to the demographics of users rather than to features of the medium itself [11, 15].

Do new media technologies like GeoGebra enable non-traditional problem-solving strategies? We demonstrate one particular problem solving approach, “messing around,” which GeoGebra facilitates quite well. We’ll describe several GeoGebra apps that encourage this approach, and explain how they were used in a hybrid, inquiry-based, community college Precalculus course. This class was “inquiry-based” in the sense that students used class time to work through scaffolded activities and make sense of mathematics on their own; there was very little teacher-led instruction in the form of lectures. The class was “hybrid” in the sense that students met face-to-face half as often as students in other sections of Precalculus, and this reduced class time was supplemented with online work. Students were asked to maintain a Wordpress blog about math. They selected two prompts from a menu of assignments each week and responded to them on their blogs. Many of these prompts used GeoGebra apps. We’ll discuss these apps, and look at a few examples of student work.

The Challenges of Learning Online

Online learning presents new challenges and limitations not encountered in face-to-face classrooms. Consider some of the challenges a student might face while working on the following problem.

For what input x does the function $f(x) = -x^2 + 5$ produce the output 4?

1. The student may not understand key terminology like “function” or “input.” In a face-to-face classroom, this student might ask their instructor or a peer for clarification, but these resources are not immediately available to the online student. At best, they might look these terms up in a book or online, or they might send their instructor an email and wait for a reply. Even if the student knows where to look, searching for these terms requires an additional investment of time and energy that’s unnecessary for the face-to-face learner.

2. A student may not know how to evaluate the function. Again, the student would have to look this procedure up, or email an instructor or a peer and wait for a reply.

3. Even if the student understands the terminology and the process of evaluating a function, they may not know where to start. How does one find the correct input? A student could guess-and-check, plugging different x’s into the function and hoping for the right y. That process is tedious, and it’s difficult to measure one’s progress. Suppose the student tries 34 different inputs and none of them produce the right output? Have they made any more progress than the student who tried only two inputs? At what point should this student give up and try something else? If the student does not already know to set up an equation and solve for x then nothing in this problem will suggest that approach.

4. In the best case scenario, the student understands the terminology, knows how to evaluate the function, and has the wise idea to set up an equation, but they still may not know how to solve the equation.

This list is not exhaustive, but the point is that none of these challenges can be overcome without looking the answer up or asking another person for help. As a result, many online classes follow an acquisition model of learning. Students watch a video and take a multiple choice quiz to ensure they’ve acquired the “correct” understanding of the material. They may learn to mimic a rote procedure, but how much are they engaged in authentic mathematical problem solving? This type of online instruction is akin to face-to-face lectures; it does not enable the type of active learning that inquiry-based classrooms aspire to engender.

Indeed, Marra and Jonassen [9] examined popular learning management systems (LMSs) like Blackboard and WebCT and concluded that because these systems “do not support the use of alternative forms of knowledge representation by learners, authentic forms of assessment, and the use of distributed tools to scaffold different forms of reasoning, the range of student learning outcomes is restricted to reproductive learning.” Blackboard acquired WebCT in late 2005 [1], and remains the most popular LMS today with roughly 32% of the market share [8]. In a 2014 blog post [7], George Kroner, a former development manager at Blackboard, observed that “while LMSs have evolved over time, they generally have the same capabilities that they had back in the late 1990s.” The world has changed a lot since 2001, but the limitations identified in Marra and Jonassen’s study have not gone away.

Jonassen et al. [6] encourage instructors to look beyond these limitations and focus on technology not as a tool to teach with, but rather as "an excellent tool to learn with." Thinking about technology as a "tool to learn with" requires an understanding of how people learn online. Mizuko Ito and fourteen other researchers [5] conducted an extensive, three-year ethnographic study of the way young people communicate and learn online, categorizing this engagement into three basic "genres of participation": "hanging out," "messing around," and "geeking out."

- **"Hanging out"** refers to friendship-driven forms of participation like sharing photos from a party on Facebook or exchanging texts with a friend.
- **"Messing around"** describes casual, interest driven experimentation with new media technologies. This category includes what Ito et al refer to as "fortuitous searching": "moving from link to link, looking around for what many teenagers describe as 'random' information" [5, p. 54]. This type of search is open-ended rather than goal-driven.
- **"Geeking out"** refers to more intense interest-driven activity with more sharply focused goals. This might include activities like maintaining a blog on a niche interest or editing a video.

It's important to note that these are "genres of participation," not genres of people. Rather than casting people into rigid categories like "digital natives" or "millennials," Ito et al acknowledge that an individual can be a "luddite" in one setting and a "geek" in another.

These researchers found that certain activities can bridge the divide from one form of participation to another. They note that "'hanging out' with friends while gaming can transition to more interest-driven genres of what we call recreational gaming" [5, p. 17]. Perhaps, "messing around" with recreational mathematics can then transition to "geeking out" about algebra?

With this in mind, we created several GeoGebra apps that serve as "distributed tools to scaffold different forms of reasoning." Each of these apps enables students to "mess around" with new mathematical ideas before transitioning to more traditional work. In the next section, we present one such app, and then explain how it enables problem-solving strategies not available in a traditional algebra problem.

Function Machines in GeoGebra

The *Function Machines* app[1] presents users with four machines and asks them to make the outputs of all four machines the same. Students "mess around," testing different inputs in each machine to see what type of output it will produce. A label on each machine also provides a formula for the output. When the student provides an input that is not in the domain of the function, the output will display a question mark. See figure 5.1 below.

This task is open-ended; there are several different correct answers, but students can begin the task with a "fortuitous search" and discover the properties of functions along the way. The app will also confirm a correct solution, presenting the student with a congratulatory message upon their success.

In many ways, this is a richer, more complex exercise than the algebra problem described above, and yet most of the challenges attributed to that problem do not occur here.

- The student does not need to know any special mathematics terminology to start working on this problem. The app presents students with a readily identifiable metaphor about machines in the real-world, and any other properties of these machines can be discovered by experimenting with different inputs.
- The student does not need to know how to evaluate a function. In fact, they can experiment with different inputs to form their own hypothesis about how the formulas for each machine relate to the output.
- The GeoGebra app automates a lot of the tedious calculations that made guessing-and-checking a poor strategy in the algebra problem above. The app also provides four functions rather than one. If a student cannot produce

[1] `http://tube.geogebra.org/m/2870887`

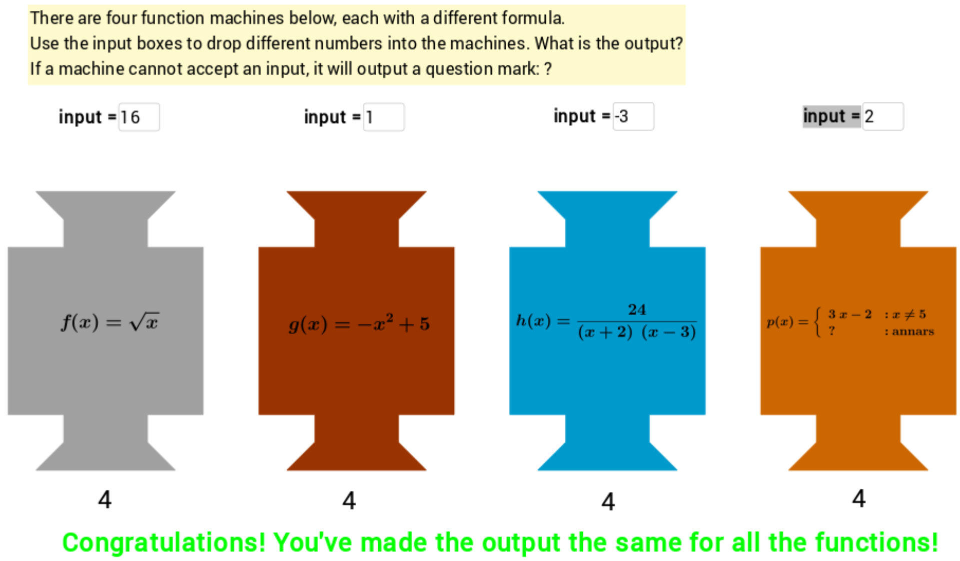

Figure 5.1: A student has successfully made all four outputs the same in the *Function Machines* app.

the desired output on one machine, they can switch to another and continue working. Thus, it's easier to get a sense of progress if the student can produce the desired output on two or three of the four machines.

This GeoGebra app is not a panacea. On its own, it will not teach students how to set up and solve the requisite equations, but it does allow students to engage in authentic online inquiry before learning more traditional techniques. By "messing around" in the app, students can begin to make meaning on their own, and this meaning will later support their understanding of the algebra. The *Function Machines* app enables "messing around" whereas the algebra problem requires some heavy "geeking out." Traditional algebra problems are still important, but "messing around" may be a necessary precursor to "geeking out."

Anatomy of "Messing Around"

By our own observations, the following five criteria seem to be necessary (if not sufficient) components of "messing around."

1. "Messing around" requires **very little domain-specific knowledge**. Students did not need any special terminology or algebraic skills to start working with the GeoGebra app.

2. "Messing around" allows for **"fortuitous searches."**[2] A student can play around with different inputs until they stumble across an answer that works. This often produces the perception for students that they are "just about to get it."

3. "Messing around" entails a **built-in "control of error"** in the sense that Montessori used this term [2, p. 80–81]. In other words, some element of the problem or app will verify the student's answers. The *Function Machines* app calculates the output of each function so the student can confirm whether or not they understand how to evaluate the functions. Likewise, when a student makes all four outputs the same, the app presents a congratulatory message, validating that they did it correctly.

[2] In the field of artificial intelligence, problem-solving is often conceived of in terms of search-algorithms [12]. In some sense, any problem-solving technique is a search (fortuitous or otherwise) for a solution. As mentioned in the introduction, Schoenfeld [17] observed that amateur problem-solvers tend to get stuck on one strategy even when that strategy is not working. Advanced problem-solvers take a more deliberate approach, switching between strategies until they find the right one. In some sense, the distinction between "messing around" and "geeking out" is as simple as distinguishing between fortuitous and deliberate searches.

4. "Messing around" is **portable**. The mobile nature of GeoGebra means that students can work on this app anywhere, thereby maintaining the casual nature of "messing around."
5. "Messing around" **does not require one's full attention**. A student can play around with the GeoGebra app while hanging out with a friend. It's a casual and therefore engaging activity that invites the transition to deeper forms of engagement when ready.

As an example of these principles at play, consider the following excerpt from a student's blog.

> I use the function app and challenged myself to see if I could make the outputs of all 4 functions the same. After 10 minutes of "Guess and Check", I figured it out. The first function needs an input of 16 to equal 4. The second function already equals 4 because the default of the input was 1. The third one was the trickiest one for me but eventually realized that the input needed to be 4. Last but not least, the fourth one needed an input of 2 to equal 4.

This student identifies their strategy as "Guess and Check," a form a "fortuitous search." However, they appear to be doing more than just randomly plugging in numbers. They note that "the third one was the trickiest one for me but eventually realized that the input needed to be 4." The student found a solution that worked for three of the four functions. They felt they were were "just about to get it" and thus felt confident enough to persist until they found an answer.

Sail Away

With these distinctions in mind, let's look at another example. The *Sail Away* app[3] prepares students to understand trigonometric functions and their connection to the unit circle. "Messing around" with this app will help students later "geek out" and solve problems such as the following.

How are the graphs of $\sin(x)$ and $\sin(4x)$ related?
What is the maximum value of the function $\sin(x)$? For what x does it achieve this maximum?

The app (in figure 5.2 below) shows a sailboat floating next to a dock. Beneath the dock is a circular dial with the instructions "rotate counter-clockwise." Rotating the dial raises and lowers the tide. The app's instructions suggest rotating the dial counter-clockwise, but the app does not forbid students from rotating the dial in the opposite (negative) direction. Students can also click on the "Animate Sail Boat On/Off" button, and the boat will begin to sail away, tracing its path as it moves. A "seconds" counter beneath the dock keeps track of time throughout the boat's animation. A wave is drawn along the water as a dashed line. Students are asked to trace the path of this wave using the sailboat and the dial. This task requires precise eye-hand-coordination and some trial-and-error.

The prompt also asks students how many times the dial goes around the circle in 8 seconds? This app is embedded in a GeoGebra book with subsequent examples, each showing waves of varying period. By focusing on the speed of the dial, students begin to realize that turning the dial faster produces a smaller period and turning it slower produces a larger period. As demontrated in figure 5.3 below, students were also able to connect the peaks and troughs of the wave with the top and bottom of the circle.

Again, consider how this app demonstrates all five criteria for "messing around."

1. **Little Domain-Specific Knowledge:** Students do not need to know anything about sine functions to begin working with the app. The context of a boat floating next to a dock is familiar to everyone and thus students can draw on their real-world experiences to reason about mathematics.
2. **Fortuitous Searches:** Through trial-and-error, students can experiment with navigating the boat. They can see how rotating the dial affects the graph and thus make sense of the scenario by playing with the app.
3. **Built-In Control of Error:** The app shows the wave as a dashed line and traces the boat's path as it moves. Students can visually compare the boat's trace with the sine function. They do not need to consult an instructor or a book to verify their answers.

[3] `http://tube.geogebra.org/m/JENexAZC`

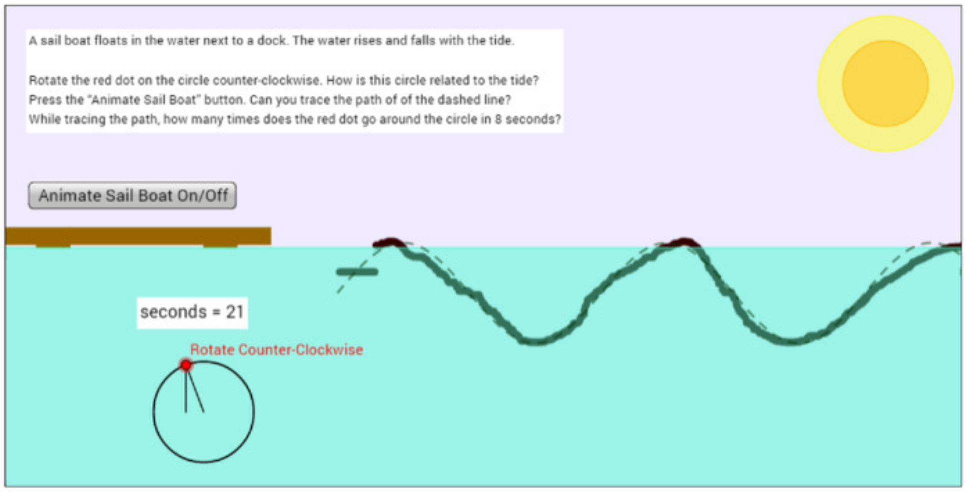

Figure 5.2: A student traces the wave by rotating the dial counter-clockwise.

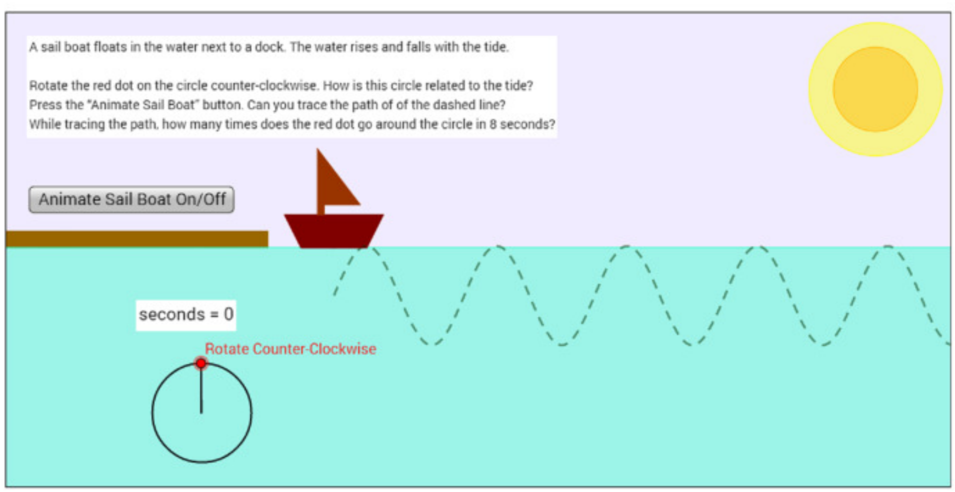

Figure 5.3: A student demonstrates their understanding of the curve's maxima and minima by observing that "at high tide the dot is located at the top of the clock (12)."

4. **Portable:** The app is portable by virtue of GeoGebra being portable.

5. **Does Not Require Full Attention:** Students can play with the dial and intermittently attempt to trace the wave while hanging out with a friend or working on other tasks.

As an example of how persistent students can be when "messing around," consider the following student blog entry titled "Annoying little Boat."

> This assignment is harder than it looks. I restarted many times because the tracing wasn't cute. In order to make the dot move you move it clock wise, it moves down until it fits 6 then it moves up until it hits 12 and then it goes down again. The highest tide is at 12 and the lowest is 6. You need to move the dot slow for you to be able to trace the ones with big gaps like the ones in sail boat 1 and 2. For sail boat 3 and 4 you need to move the dot faster. If you move it slow then you will create big waves and if you move it fast it will create tight waves.

This student is clearly irritated with the "annoying little boat," and yet they continue to work on the problem precisely because it enables them to "mess around" until they find a suitable solution. Likewise, the student is aware of the problem's built-in control of error as they "restarted many times because the tracing wasn't cute."

Note that this student overlooked the app's instructions and chose to move the dial clockwise. This also demonstrates some of the limitations of "messing around." The student is able to develop a surprising level of understanding about the trigonometry of the unit circle, but without explicit instruction, they will not be aware of the convention of tracing positive angles in the counter-clockwise direction. On it's own, this app will not teach the student formal trigonometry, but it does allow the student to start making sense of the mathematics on their own without a deliberate strategy in mind and without immediate access to their instructor.

Additional examples of apps that support "messing around" include the *Composition of Functions* app[4], the *Exponents Game*[5], and the *Radians Game*[6]. We suggest the reader "mess around" with each of these apps, and consider for themselves the extent to which they do or do not align with each of the five criteria identified above.

Why GeoGebra?

We've provided several apps designed in GeoGebra, each of which enables students to "mess around" with new mathematical ideas before engaging in more traditional classroom work. Each of these apps could have been developed without GeoGebra. They could have been programmed by hand in Javascript or constructed using a platform like GDevelop[7]. Programming apps by hand is time-intensive and requires specialized knowledge that most teachers do not have. GDevelop requires a smaller time investment and virtually no coding, but it does not have the same built-in mathematical functionality as GeoGebra.

The advantage of GeoGebra is that it provides a quick and easy way to build one-off apps for specific mathematical problem-solving situations. Educators can eschew monolithic tools like Blackboard in favor of their own DIY creations much along the lines of what Jim Groom has termed "edupunk" teaching [3]. With a critical mass of creators, this could be a powerful pedagogy, enabling new problem-solving strategies for a new media landscape. As we've demonstrated, new media provide opportunities for students to engage in mathematical thinking without developing a deliberate strategy or a clear end-goal. By "messing around," students can engage in authentic mathematical problem-solving without appealing to their instructor every time they veer off the beaten path. This more casual form of engagement can form a gateway to more serious "geeking out" such as traditional homework problems and face-to-face classroom instruction.

4 `http://tube.geogebra.org/m/xKCGjZxj`
5 `http://tube.geogebra.org/m/3143617`
6 `https://www.geogebra.org/m/VWcWKR9E`
7 `http://compilgames.net/`

REFERENCES

1. J. Clabaugh, *Blackboard buys competitor for $180M*, Washington Business Journal, October 12 (2005). URL: http://www.bizjournals.com/washington/stories/2005/10/10/daily20.html.
2. W. Crain, *Theories of Development: Concepts and Applications*, Prentice Hall, 2011.
3. N. DeSantis, *Self-Described â^ŸEduPunk' Says Colleges Should Abandon Course-Management Systems*, The Chronicle of Higher Education, February 26 (2012). URL: http://chronicle.com/article/self-described-edupunk-says/130917
4. M. L. Frederick, S. Courtney, and J. Caniglia, *With a Little Help from My Friends: Scaffolding Techniques in Problem Solving*, Investigations in Mathematics Learning, 7 (2014), no. 2, 21-32.
5. M. Ito, S. Baumer, M. Bittanti, D. Boyd, R. Cody, B. Stephenson, H. Horst, P. Lange, D. Mahendran, K. MartÃ›nez, C. J. Pascoe, D. Perkel, L. Robinson, C. Sims, and L. Tripp, *Hanging Out, Messing Around, and Geeking Out: Kids Living and Learning with New Media*, MIT Press, 2009.
6. D. H. Jonassen, J. Howland, J. Moore, R. M. Marra, *Learning to solve problems with technology: a constructivist perspective*, Merrill, 2003.
7. G. Kroner, *Does Your LMS Do This?*, Edutechnica, January 7 (2014). URL: http://edutechnica.com/2014/01/07/a-model-for-lms-evolution/.
8. ListEdTech, *LMS Providers' Market Share by Implementation Year*, November 23 (2015). URL: http://listedtech.com/lms-providers-market-share-implementation-year/
9. R. M. Marra and D. H. Jonassen, *Limitations of online courses for supporting constructive learning*, Quarterly Review of Distance Education, 2 (2011), no. 4, 303-317.
10. J. Mason, L. Burton, K. Stacey, *Thinking Mathematically*, Pearson, 2010.
11. A. Nicholas, *Preferred learning methods of the millennial generation*, The International Journal of Learning, 15 (2008), no. 6, pp. 27-34.
12. N. J. Nilsson, *Problem-solving methods in artificial intelligence*, McGraw-Hill, 1971, pp. 65-68.
13. M. Montague, *The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities*, Journal of learning disabilities, 25 (1992), no. 4, 230-248.
14. G. Pólya, *How to Solve It: A New Aspect of Mathematical Method*, Princeton University Press, 2014
15. M. Prensky, *H. sapiens digital: From digital immigrants and digital natives to digital wisdom*, Innovate: journal of online education, 5 (2009), no. 3, art. 1. URL: http://nsuworks.nova.edu/cgi/viewcontent.cgi?article=1020&context=innovate
16. M. Santos-Trigo, L. Moreno-Armella, *The Use of Digital Technology to Frame and Foster Learners' Problem-Solving Experiences*, Posing and Solving Mathematical Problems (P. Felmer, E. Pehkonen, and J. Kilpatrick, eds.), Springer International Publishing, 2016, pp. 189-207.
17. A. H. Schoenfeld, *Expert and Novice Mathematical Problem Solving. Final Project Report and Appendices B-H*, 1982. URL: http://eric.ed.gov/?id=ED218124